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THE MATHEMATICS TEACHER

Volume XXVI Number 5

Edited by William David Reeve

The Influence of the College Entrance Examinations on the Teaching of Secondary Mathematics*

By L. H. WHITCRAFT

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A FEW DAYS AGO your speaker was privileged to hear a lecture by Strickland Gillilan, one of America's outstanding humorists. One thing emphasized particularly in that lecture was that facts of today will not be facts tomorrow. During the past few years this has been repeatedly demonstrated. The world has been changing so rapidly that the most of us have found it utterly impossible to make satisfactory adjustments. The data which I shall report in this paper were facts only a short time ago but I cannot vouch for them all as true statements of existing conditions today.

During the years 1930 and 1931 information was gathered concerning the influence of the College Entrance Board examinations and requirements upon the teaching of secondary mathematics. At various times statements have been made concerning influences which the examinations have had. Some of these statements were made by persons who spoke with authority upon the subject; for example, two years ago William Betz, now president of the National Council of Teachers of Mathematics, addressed the Council

* Read at the annual meeting of *The National Council of Teachers of Mathematics* at Minneapolis, February 25, 1933.

at its annual meeting in Detroit. He brought to the attention of those who were present at that meeting the position the College Entrance Examination Board and the Board of Regents of the State of New York occupied and their opportunity to influence the mathematics of the secondary school. Figures were introduced to show the increasing number of candidates coming to the Board examinations from an increasing number of high schools from all parts of the United States. In 1930, 1115 public and 843 private secondary schools, or a total of 1959 schools contributed 23,478 candidates to the June examination of the College Entrance Board. This group of candidates wrote 72,337 books of which 20,841 or slightly more than one-fourth were written on the subject of mathematics. Obviously the number of contacts the Board had with the secondary schools was sufficient to have some effect on the mathematics of those schools. The question is, did it have any influence upon the mathematics of the secondary schools? If so, to what extent?

To answer this and other questions raised concerning the effects of the College Entrance Board requirements and examinations data were gathered from the heads of the departments of mathematics in both public and private secondary schools and from superintendents of schools which send candidates to the Board examinations; from statements of authors of textbooks in secondary mathematics; from state and city courses of study; from an analysis of the questions used in the annual June examinations of the Board; from specialists in the teaching of mathematics; from statements of curriculum specialists and general educators; and from the annual reports of the secretary of the College Entrance Examination Board.

The one hundred twenty-three teachers of secondary mathematics who returned questionnaires in this study were practically unanimous in the belief that the annual examinations of the College Entrance Board had some influence on secondary school mathematics. More than 50% of these teachers believed this influence was great. Very little difference was found in the percentages of teachers from the public schools and those of the private schools regarding the amount or extent of this influence. The returns from forty-nine superintendents out of sixty-five who were mailed questionnaires showed that nineteen of them thought the Board's examinations exerted great influence; twenty-eight, some

influence; one, no influence; and two failed to signify what they thought.

TABLE 1

Opinion of One Hundred Twenty-Three Secondary Mathematics Teachers and Forty-Seven Superintendents on the Amount of Influence the College Entrance Examination Board Examinations have upon the Mathematics of the Secondary School

Degree of influence	No. Teachers			No. Supts.
	Public schools	Private schools	Total	
Great	35	33	68	18
Some	30	23	53	28
No influence	1	1	2	1
Total	66	57	123	47

An effort was made to find whether the superintendents of schools in eastern states differed on the question of degree of influence of the College Board examinations from those superintendents of schools in the central and western states. By this grouping there were 31 reports from Eastern schools and 18 from the schools in the central and western states. 42% of the superintendents from eastern schools claimed great influence was exerted by the Board examinations against 28% of the superintendents from the Central and Western States. This would seem to indicate that the Eastern schools were affected more by the College Entrance Board examinations than the schools farther west.

Textbooks in secondary algebra and geometry were examined for statements concerning the attempts of the authors to satisfy the requirements of the College Entrance Board.

TABLE 2

Requirements which the Authors of Textbooks in Secondary Algebra and Geometry, Published Since 1920, Claimed to have Fulfilled

Text	Number examined	Col. Ent. Ex. Board	National Committee	Board of Regents
Algebra	22	15	11	6
Geometry	30	22	20	9

Out of twenty-two elementary algebras examined that have been published since 1920, fifteen contained statements by the authors

that they fulfilled the requirements of the College Entrance Board; eleven that they satisfied the syllabus of the National Committee on the Reorganization of Secondary Mathematics; and six that they fulfilled the requirements of the Board of Regents of the State of New York.

Thirty textbooks in elementary geometry, published since 1920 were examined for statements concerning the fulfillment of the requirements of the College Entrance Board. Of this number twenty-two, according to the authors of these texts, fulfilled the requirements of the College Entrance Board; twenty, satisfied the syllabus of the National Committee; and nine fulfilled the requirements of the Board of Regents. In each of these secondary subjects the College Entrance Examination Board received greater recognition than the National Committee on the Reorganization of Secondary Mathematics.

City and State Courses of Study were examined for evidence of College Entrance Board influence on the mathematics of the secondary school. Here were found references to the College Entrance Board requirements and examinations in statements of aims for advanced algebra and geometry; in the definite requirements of these subjects; in review courses organized to prepare students particularly for the Board examinations; in the list of reference material for teachers; and in the suggestions to teachers of geometry that they emphasize the theorems which are starred in College Board list of theorems.

Statements of persons prominent in the field of education brought additional evidence concerning the influence of the requirements and examinations of the College Entrance Board. Professor Hedrick, in an address before a group of graduate students at Teachers College, Columbia University during the summer of 1930 stated that College Entrance examinations in mathematics have kept formalism in the class room; that teachers realizing that their success depended upon the success of their pupils on the examinations spent much time in drill upon the types of exercises given in the examinations.

Dean McConn of Lehigh University, in a letter to your speaker made this statement; "I do believe that the general effect of College Entrance examinations is to turn the work of the senior year, in the private preparatory schools, especially, into a cramming course devoted to an intensive review based upon the questions

given in previous years by the College Entrance Examination Board or the examining colleges. Such a review is undoubtedly effective for the purpose of passing examinations, but I believe that any competent teacher can accomplish more in the way of giving real grasp, understanding, and appreciation of any subject if he is free to work with his mind on such purposes rather than with his eye on specific examination questions."

Each of these quotations point to certain undesirable influences of the College Board examinations. Permit me to quote from men who view the work of the Board in a different light. Nicholas Murray Butler, President of Columbia University, made the following statement back in the year 1913, "In my judgment, the College Entrance Examination Board is by far the most useful single constructive force that has ever come into the field of American secondary and collegiate education."

Wilson Farrand, in an address delivered at the dinner in celebration of the twenty-fifth anniversary of the founding of the College Board made this statement, "It is my deliberate conviction that the strongest factor in the improvement of our secondary schools in the last twenty-five years is to be found in the standards set and maintained by forward and upward looking teachers working through the agency of the College Entrance Examination Board."

The January issue of the Mathematics Teacher this year contains an article by Dr. Longley of Yale University on "The Function Concept in Algebra." Among other things Dr. Longley says, "The work of the National Committee accomplished important results but it suffered from two handicaps. It lacked authority to enforce any of its ideas and its existence was temporary. Fortunately, to a certain extent at least, these two defects were remedied by the College Entrance Examination Board. There are some undesirable effects of the influence of the Board examinations on secondary school teaching which no one deplores more than I do, but I believe that the beneficial outweigh the deleterious effects and an example is the result of the present definitions of requirements in mathematics. These definitions were almost exactly a restatement of the results reached by the National Committee. The effect was that some schools were forced to follow more modern ideas in their teaching of mathematics. I feel sure that some of our best known schools would now be teaching algebra exactly as it was

taught twenty years ago if they had not been compelled to change by the College Board Requirements.

Some features of the progress represented in the report of the National Committee are readily enforceable by examination. An obvious example is the introduction of a certain amount of trigonometry into elementary algebra. Other features are not so easily forced upon an unwilling teacher."

One could cite a great number of statements made by nationally known educators to show that they are convinced that the College Entrance Board Requirements and Examinations have had and do have at this time much influence upon the mathematics of the secondary schools but time forbids.

From all of these various sources which have been mentioned the data shows conclusively that the examinations of the College Entrance Board have had much to do with the mathematics of the secondary schools. Our answer then to the question stated earlier, did the College Entrance Examination Board have any effect upon the mathematics of the secondary school is, it did and it continues to have much influence upon those schools which furnish candidates for the Board examinations and some influence upon practically all other schools.

Effects of the College Board Examinations

Our second question is "How have these examinations of the College Board affected secondary mathematics?" Some of the effects have already been indicated.

Proctor in 1925 found the interests of those secondary school pupils not going to college, estimated to be about 75% of all secondary school pupils, subservient to the interest of the 25% or those going to college. Boardman, in 1926, found that a large majority of students who entered college took secondary mathematics not from choice or interest but only to satisfy a college entrance requirement. Mr. Nelson Snyder, Principal of the South Side High School, Ft. Wayne, Indiana, told your speaker last spring that a large number of students in the South Side High School were enrolled in classes in algebra and geometry simply be-

[1] School and Society, Vol. XXII, Oct. 1925, pp. 441–448.

[2] Boardman, Chas. W. The Preparation of High School Graduates of Minneapolis and St. Paul for College Entrance, unpublished Master's Thesis, Teachers College, New York, 1926.

cause they hoped to enter college sometime later, and not because they had any interest or ability along mathematical lines. In other words college entrance requirements determines the type of mathematics most high school pupils take. Your speaker does not question the right of any college to set up definite entrance requirements, even though it means that a particular line of work must be followed in the secondary school but he does question the advisability of forcing this type of work on those pupils who will never enter college.

One-fourth of the teachers who returned questionnaires in this study claimed that the College Entrance Board examinations interfered with the best teaching of secondary mathematics by forcing a curriculum upon the secondary schools; two-fifths or 40% of this group of teachers claimed the examinations permitted but little freedom in the choice of materials and content of the courses of secondary mathematics against 25% who claimed that the examinations permitted much freedom in the choice of materials and content. An idea of the amount of freedom these teachers felt they had is shown in the following; 25% of the teachers had desired at some time to do work of a creative nature in their mathematics classes but found it impossible because of coming Board examinations. Others claimed that they had at different times wished to make changes in their mathematics courses but could not because of the examinations.

Some of the things these teachers wished to do but could not because of the Board examinations were: (a) give much applied work in geometry; (b) to teach a course in geometry in which the work was developed historically; (c) to do much experimental work in geometry; (d) to develop a course in geometry which included plane, solid, and analytical geometry; (e) to teach geometry by the laboratory method; (f) to include a unit in trigonometry in the geometry course; (g) to develop a course in geometry in which the class made the originals; (h) to introduce calculus in geometry; (i) to introduce field work in geometry and trigonometry (j) to give a course in surveying; and (k) to develop and give courses in general mathematics.

In this connection I wish to quote from three teachers. Teacher No. 1 said: "The Board certainly does not interfere with my own research work. As for the pupil, the large number of problems and exercises given him challenge all the creative ability he may possess."

In contrast to this statement was a statement from teacher No. 2, who said: "One cannot bring into the recitation the meanings or practical applications of mathematics because so much time must be spent in drill in manipulation."

Teacher No. 3 in commenting upon the freedom the teacher has in the choice of materials and content in secondary mathematics said: "The College Entrance Board examinations certainly prescribe almost absolutely the curriculum in the mathematics of a high school which prepares any number of pupils at all for the examinations. The curriculum which it prescribes is not obsolete by any means, but there are many other interesting topics which a teacher might prefer to teach but which training for the examination leaves no time to pursue. For example, Graphing. I should like to spend three weeks on statistical graphs, but cannot possibly do it and prepare my pupils for the algebra examination. For a new teacher, especially, the College Board examination is a sword of Damocles suspended over her head. Cramming pupils for examinations is not teaching.

I do not feel that the effect is wholly bad, but I do feel that it gives teachers a very narrow road in which to travel and tends to a mechanical proficiency which is not good." The superintendent of the school in which this teacher was teaching stated that he concurred to all the teacher had said and then some.

The superintendent of schools in one of our great industrial cities of the middle west made this comment: "Even though possibly no more than 1% of the graduates of certain high schools take the examinations the principals and mathematics teachers point the mathematics courses toward the examinations."

Many of the teachers indicated also that the examinations forced the attention of the teachers and pupils upon the examinations and in so doing hindered progress in secondary mathematics. Mr. Betz told us two years ago, that in certain schools the examinations of the College Board or of the Board of Regents became the real course of study for the last year of the secondary school course. Dean McConn stressed this same point in the letter to which I have already referred.

An attempt was made in this study to find out the method by which pupils were prepared for the Board examinations. According to the reports from 123 schools, seventy-seven prepared their pupils for the Board examinations in regular classes; (Regular here

meaning no prerequisites other than course prerequisites) eleven in classes composed entirely of pupils preparing to take the Board examinations; two by tutoring; seventeen in both regular classes and in classes composed of those pupils preparing for the examinations; thirteen, in regular classes and by tutoring; and the remaining three schools by all three methods.

TABLE 3

Method of Preparing Pupils in One Hundred Twenty-Three Secondary Schools for the College Entrance Board Examinations

Method	Public Schools	Private Schools	Total Number
a. Regular classes	35	43	77
b. Classes composed entirely of candidates for exams.	7	4	11
c. Tutoring	2	0	2
d. Both (a) and (b)	11	6	17
e. Both (a) and (c)	9	4	13
f. All three methods	3	0	3
Total	67	56	123

None of the private schools reported in this study prepared students for the Board examinations by tutoring. Where preparation was made other than in the regular classes various methods were employed. For instance, some schools spent as much as one semester reviewing for the examinations. Others indicated that from one to two periods each week were spent in reviewing depending upon the needs of those who were to take the examinations and the number of subjects carried. In most cases the review classes met outside the regular school hours under the supervision of one of the mathematics teachers. At Culver Military Academy, Culver, Indiana the last semester of the senior year is given over to review. This is probably the case in many other institutions. A statement was made by the General in charge of that academy that if it were not for the College Entrance Board examinations Culver could give a boy a real education.

The annual questions of the College Entrance Board were used by all except eight institutions in the preparations for the examinations. Forty-six of the 123 schools used the questions to supplement the text; twenty-nine used them only in the preparation of pupils for the examinations; forty schools used them both to sup-

plement the text and to review for the examinations; while eight did not use the questions of the College Board in any way.

In order to find the particular topics in algebra and geometry that received greater or less emphasis because of the College Board examinations a list of topics in each of these two subjects was submitted to the teachers and they were asked to designate by checking the topics which received greater emphasis because of the Board examinations; also those topics which received less emphasis. The reports from these teachers indicated that all of the topics of algebra except the highest common factor, the least common multiple, polynomials, and ratio and proportion received greater emphasis. The two topics which received the greatest emphasis were logarithms and the formula. In just two instances were there disagreements between the public school teachers and those of private schools. The public school teachers claimed they put slightly more emphasis on polynomials and ratio and proportion while the private school teachers indicated they put slightly less emphasis on these two topics. To the speaker this agreement seems rather significant.

In the field of geometry the teachers of the two types of schools were in agreement on all points considered. The reports indicated that greater emphasis was placed on the starred theorems, of the Board list, on locus problems, on originals requiring proof, on originals requiring solutions, and on construction exercises while less emphasis was placed on definitions and the unstarred theorems of the Board list of theorems.

Analysis of Board Questions

An analysis of the College Board questions used in the subject of elementary algebra in the Annual June examinations showed that on basis of frequency of occurrence the topic of factoring would be rated as the most important topic. Twenty-one percent of all the exercises given on the Board examinations over a period of thirty years were in factoring. How can one justify so much emphasis upon this topic? At no time since the establishment of the public high school in 1820 have textbooks in elementary algebra devoted on an average of more than fifteen percent of the exercises to factoring. During the past decade, 1920 to 1930, textbooks devoted on an average, less than ten percent to exercises in factoring.

Simultaneous equations ranked second in importance if judg-

ment be based upon the number of exercises given in the examinations. In the light of modern trends this topic is fairly obsolete yet sixteen percent of the examinations have been given to it. The topic of radicals ranked third with a percentage slightly less than twelve. The three topics factoring, simultaneous equations, and radicals made up practically fifty percent of the examinations up to and including the year 1930. Since 1930 there has been some improvement along this line, however.

Just one exercise requiring the division of a polynomial by an expression of two or more terms was included in the examinations for Mathematics A1 between the years 1904 and 1930 and but two exercises in ratio and proportion between 1911 and 1930. Here appears a reason for the teachers giving less emphasis on these two topics.

In 1926 two new topics were introduced into the elementary algebra examinations. These topics were graphs and trigonometry. Each of these topics has held a prominent place on the annual examinations since their introduction. Teachers said they gave greater emphasis on these topics because of the examinations.

Although the College Entrance Examination Board eliminated much of the obsolete material from its syllabus in 1923 there still remains some obsolete material in the examinations. The type of complex fraction included in the examinations in elementary algebra, Mathematics A1, was not changed during the first thirty years of the Board examinations. These complex fractions are obviously more difficult than any the average student will meet in the most difficult formula with which he will likely work. In the light of modern trends in education the type of complex fraction which the College Entrance Board examinations tends to perpetuate is obsolete.

Geometry

An analysis of the questions used over a period of thirty years in the subject of geometry shows definitions no longer are included in the questions for plane geometry and just a few have been included during recent years in solid geometry.

Exercises having to do with loci were first introduced in the annual June examinations in geometry in the year 1906. Since that date there has been a locus problem in the examinations in practically every year. An article appeared in the November issue of

the Mathematics Teacher, 1929, entitled LOCOPHOBIA. The author of that article called attention to the fact that for the years 1921 to and including 1928 ten percent of the College Board examinations in plane geometry was given to locus problems. According to the author of that article this percentage was too high. It was not as high, however, as it was during the period 1911 to 1918 when eight out of forty-eight questions or one-sixth was given to locus problems. Teachers who coach or prepare pupils for these examinations know the percentage of questions given over to many of these topics and they permit this knowledge to influence the amount of emphasis given to the various topics. One cannot blame them for doing this since it is true that the degrees of efficiency of teachers and of classes in the same or different schools have been compared on the basis of relative scores made on the Board examinations.

TABLE 4

Number of Theorems and Exercises Given in the Annual June Examinations of the College Entrance Examination Board in Plane Geometry, Mathematics C, from 1900 to 1930

Decade	Theorems		Exercises		
	Starred	Unstarred	To prove	To solve	To construct
1921–1930	20	1	16	23	4
1911–1920	10	12	25	21	12
1901–1910	13	16	45	29	15

A check of the theorems used in the annual June examinations in plane geometry from 1901 to 1930 against the 1923 College Board list showed that there had been a decrease in the number of "unstarred" theorems and an increase in the number of starred theorems. Eight of the starred theorems were not included in any of the examinations, while six of the starred theorems had appeared as many as three times each. Twenty out of the fifty-eight unstarred theorems had been included at least once while two of the unstarred theorems appeared four times each.

An examination of the questions of solid geometry for the same period showed practically the same tendencies as for plane geometry. More and more emphasis was placed upon the starred theorems and less and less on the unstarred theorems. Each of the twenty-two starred theorems appeared at least once during the

period of thirty years and two of these appeared five times each. Fifty of the seventy unstarred theorems were not included in any of the examinations yet one of the unstarred theorems appeared as many as five times.

In both plane and solid geometry there was found to be a tendency to include fewer originals to prove and to construct. In plane

TABLE 5

Number of Theorems and Exercises Given in the Annual June Examinations of the College Entrance Examination Board in Solid Geometry, Mathematics D, from 1900 to 1930

Decades	Theorems		Exercises		
	Starred	Unstarred	To prove	To solve	To construct
1921–1930	20	3	13	30	1
1911–1920	18	10	14	28	1
1901–1910	15	20	27	29	4

geometry the number of originals to solve was decreasing while in solid geometry there appeared no change. Textbooks have not followed the examinations in number of originals for a check on the number of originals given in textbooks published during the decade 1921–30 showed more than twice as many as were included during the decade 1901–1910.

I could continue to cite other instances of the effects of the College Entrance Board examinations on the mathematics of the secondary schools if time permitted. Briefly stated the message which this paper aims to convey is: first, the effects of the examinations of the College Entrance Examination Board are far-reaching. Practically no schools in this country are entirely free from their influence; and second, that progress in the teaching of secondary mathematics has been slowed down in many schools because of the following; (a) excessive amount of attention required in reviews and drills for the examinations; (b) a forced curriculum upon a large percentage of pupils whose interests and abilities lie in other fields; (c) undue emphasis placed upon certain topics at the expense of an understanding appreciation, and application of other topics equally important; (d) retention of formalism in the mathematics classes; (e) the pressure for time making it impossible for many teachers to do work of a creative nature; (f) retention of some types of exercises which at the present time are practically

obsolete; and (g) utilization of time which could be spent more profitably by many students.

I do not wish to imply that the examinations of the College Entrance Board are wholly bad or that the influences are all undesirable. They are not. In the main the examinations are good. Much improvement has already been made on the examinations yet there is room for more improvement. The frequency of certain questions should depend somewhat upon the importance of the topic as well as suitability for examination purposes. Obviously this has not been considered in the past to the extent that it should be. Whatever improvement is made will be reflected soon after in the teaching, especially in those schools whose graduates are candidates to the Board examinations. So long as an examination in subject matter is the chief agency in the selection of college students, teachers will employ all means at their command to prepare their students for the examinations. This means excessive drill and much cramming. Better trained teachers will help the situation but will not correct it. Grouping of students in mathematics according to whether or not they intend going to college as the City of Baltimore has done will relieve the undesirable effects in many instances. These along with courses of study made particularly to fit the needs of the various groups of students in the different communities are things which the schools themselves can do to relieve some of the ill effects of the College Entrance Board examinations on the mathematics of the secondary schools.

One of the sixty outstanding educational books of 1932 in the list selected by the Enoch Pratt Free Library in Baltimore and prepared for the National Education Association is The Seventh Yearbook of the National Council of Teachers of Mathematics. Have you seen it? It can be secured for $1.75 postpaid from The Bureau of Publications, Teachers College, Columbia University, 525 West 120th Street, New York City.

The Ideal Preparation of a Teacher of Secondary Mathematics from the Point of View of an Educationist*

By WILLIAM C. BAGLEY
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IT WOULD SCARCELY be profitable, I think, to place too strict a construction on the term "ideal" as used in the title of this paper. In fact I have been so continuously confronted with the handicapping conditions that must be met in preparing teachers that I should hardly know what to do or to say if these conditions were suddenly removed. For example, there is always the matter of the time that can be given to preparation; there is always the type of person whom we may expect to make the preparation; there are always large gaps in our knowledge as to the best ways in which to select and organize materials; and in the preparation of a high-school teacher there is the practical need of providing training that will enable him to teach in at least two fields of specialization.

We shall start, then, by assuming certain conditions: (1) a four-year program following high-school graduation; (2) a group of students who reflect the top quarter of the high-school graduating class in respect of scholarship, who in the high school have completed at least three years of secondary mathematics with a measure of success which indicates that they are mathematically-minded, and who are genuinely interested in the problem of teaching high-school boys and girls; (3) a teachers' college well equipped with mathematical materials, including instruments suited to demonstrate the various applications of mathematics, and well equipped too with laboratory schools of secondary grade; (4) a well trained staff all of the members of which satisfy the standards of scholarship demanded of teachers in the best institutions of collegiate grade, all of whom are interested in the problems of mathematics teaching in the lower schools, and at least half of whom are equipped with the training in educational theory, educational psychology, and educational measurements that is essential to a relatively ex-

* Read before the annual meeting of *The National Council of Teachers of Mathematics* at Minneapolis on February 25, 1933.

pert treatment of the problems involved in adapting mathematical instruction to learners on the lower levels and to learners of varying abilities. At least one member of the staff should have a special interest in and knowledge of the history of mathematics as such and the history of mathematics as an educational discipline. We shall also take it for granted that the student will have one field of specialization in addition to mathematics.

Under these conditions, I should divide the four years' work somewhat as follows: (1) To the two fields of specialization I should expect that the student would give approximately half of his time. Assuming that this half would be divided equally between the two fields of specialization, mathematics would have approximately the equivalent of thirty semester hours. I shall refer to courses in the two fields of specialization as *professional content courses*. They will include the equipment directly needed in the secondary teaching of the subject and the equipment indirectly needed. By the latter I mean the additional courses in the specific field that are necessary to give to the teacher a thoroughly firm grasp of his teaching materials. The specific content of such courses in mathematics would be determined primarily by the mathematics staff.

(2) Approximately forty semester hours would be given to what I shall call *professional background courses*. I mean by these, courses outside of his two fields of specialization that add desirable elements to his equipment as a teacher. This is on the assumption that wide horizons and a rich culture are fundamental in the professional equipment of every teacher. It is to be noted that I call these not cultural courses but professional background courses because I wish to emphsize my belief that the educational equipment of every teacher, and especially of every teacher in the elementary and secondary schools, should not be that of the narrow specialist, but rather that of the teacher-scholar who knows his own subject in its broader relationships and in its broader applications. In short, instead of multiplying what we are calling *integrated courses* crossing many subject-matter lines, I should prefer to safeguard the essential unity within the field but to have the teacher so well equipped that he can point out to the learner the relationships between his field and the fields of his colleagues. This will effect the end that the integrationists have in mind, I believe, and at the same time prevent the catastrophe that befalls the learner when the fusion courses become confusion courses.

These background courses should be quite distinct from the courses offered for specializing students. I should hope, for example, that teachers of history, teachers of English, teachers of art, primary teachers, intermediate-grade teachers, and all other teachers not specializing in mathematics would have a course offered by the mathematics staff dealing with the materials of college mathematics for the non-specializing student. Many of these background courses will be of the survey variety. All teachers except those specializing in the natural sciences should have a survey course in the physical sciences. I think of this as covering a year and a half or nine semester hours and including a survey of astronomy, physics, chemistry, and geology. This would be in addition to a full year of biology, which I should require of all teachers. Science teachers, of course, would have much more elaborate courses in each of the physical and biological sciences.

Perhaps I can make my conception of the professional content and professional background courses a little clearer by reference to the equipment that I should expect each to give. In the fields of his specialization, the student should make the acquaintance not only of the treatise-literature of his field; he should also know the monographic literature, both in some important corner of present-day research and through the classic monographs in his field; he should know the principal technical journals reporting discoveries first-hand and furnishing authoritative reviews of additions to the literature of his field; even as a student he should be encouraged to subscribe for one or more of these journals but whether he subscribes or not he should be encouraged to read them religiously as they come from the press. In his junior and senior years, the acquaintance with the periodical literature of his field should extend to the journals of at least one non-English-speaking country. To this end I have long recommended that in every four-year curriculum for the preparation of teachers two years of foreign-language study be required. If the student already has a reading mastery, say, of French, then the two years should be spent in gaining a reading mastery of German or of any other language in which important contributions to his field are published.

Let us now take the non-specializing student. What will the professional-background courses in subjects that he will not teach do for him in such concrete terms as those just used with respect to the specializing student. I would wish such courses to give to the

non-specializing student something more than a speaking acquaintance with the major fields of scholarly endeavor, but quite obviously the acquaintance cannot go so deep as the acquaintance of the specializing student must go. Reduced to such terms as I have just used with reference to the latter, I should say that for each of the major fields of knowledge, the non-specializing student should have read and assimilated one or more books representing the less technical treatise-literature of the field; should know the names and something of the contributions of those who to-day speak with the most highly respected authority in the field as well as the names and contributions of the outstanding scholars of the past; should have a reasonable understanding of the methods of investigation used in making new discoveries in the field; and should know in what relatively non-technical journals such discoveries are likely to be reported.

My suggestion is that, between the professional content courses on the one hand and the professional background courses on the other, the prospective teacher should have a fairly thorough acquaintance with two fields of specialization (in both of which he has had good basic work on the secondary level) and something more than a speaking acquaintance with other major fields of human endeavor. The program would be organized on the basis of prescriptions rather than on the elective plan, but provisions should be made for adjustment to individual needs, especially in the matter of insuring contacts with major fields in which the student may be especially deficient. It is assumed that his basic interests will be in his fields of specialization.

New College for Teachers, an undergraduate subdivision of Teachers College, Columbia University, starts with three innovations which may well be considered as worth a trial in other professional schools. One of these is a series of seminars which it is hoped will integrate the work done in separate subject-matter courses; a second innovation is the requirement of a period of study and travel in Europe, presumably during one of the summers; and the third is a similar period devoted to actual employment in an occupation other than teaching—factory work, social-center work, work in department stores, work on a farm or in the transportation service. These requirements will aim to provide another type of background for the teacher's professional work—something that will supplement or complement his classroom and laboratory experiences.

So far the discussion has concerned what may be called the sub-

stantive equipment of the teacher. Woven intimately with this, especially in the professional-content courses, will be the most useful materials now usually taught in separate courses in education. My own theory, from which there are many dissenters, is that the number of these detached courses can be and should be markedly reduced on the ground that the teacher's needs can be more adequately met if the problem of selecting subject-matter for the lower schools, the problem of adapting these materials to the capacities of the younger learners, and the problem of relating one's own subject to other subjects are considered in the subject-matter courses of the professional school rather than in the so-called professional courses. The older terminology itself is absurd, for if any courses are truly professional for the prospective teacher it is those that provide him with his stock in trade. There are rules of teaching and standards of good practice in teaching that can be applied to practically all subject-matter fields, but it is best I think for the prospective teacher to get his initial acquaintance with these in his own field and in the observation of the work done by competent teachers in his own and other fields, and come later to a brief course which treats these rules and standards in the light of educational theory. The quite opposite policy still governs pretty generally in our professional schools for teachers and in the liberal-arts colleges which preparate teachers. The student receives his substantive materials in so-called academic courses, often with little reference to the professional use that he will make of them; while he receives instruction in generalized teaching procedures from educational generalists some of whom have at most only a speaking acquaintance with his own subject field and many of whom have a distinctly doctrinaire attitude.

For the four-year undergraduate professional school, and for teachers of such subjects as mathematics, history, English, and the like, I should limit the detached courses in education to not more than ten or possibly twelve semester hours exclusive of the student's contacts with the laboratory schools. In so far as the latter are concerned with the teaching problems in his fields of specialization, they should be regarded as part and parcel of his professional-content work, and the supervision of his student-teaching should be looked after in large part by representatives of the subject-matter staff. In the laboratory high schools and on the upper levels of the laboratory elementary schools, critic teachers should be closely affiliated with the corresponding subject-matter groups in the college.

The contacts of the prospective teacher with the laboratory schools should begin very early in his course of pre-service training. They should include systematic observation of the best teaching, and if possible observation of teaching procedures that represent the best from the standpoint of each distinct school of educational theory. These observations should be closely related to the professional-content courses as well as to the courses in education. They should be in part replaced and in part supplemented by definite assignments of responsible duties in one of the laboratory schools, and they should culminate in a brief period—say nine weeks—of fairly complete responsibility for the conduct of a class through one or more units of work.

I use the term laboratory school in place of practice school or training school because I wish to emphasize my conviction that the student-teacher in a professional school is not to be regarded as an apprentice or a cadet, but rather as a student. It is not to be expected that this initial practice of his art will give him all of the skills and insights that he will need. It is rather to be hoped that his contacts with the laboratory schools will give point and meaning to his courses in subject-matter and in educational theory, familiarize him with the standards of good practice, and furnish a start in the acquisition of skills and insights.

It goes without saying, of course, that no schedule of courses, activities, and other stated requirements will work automatically to produce an efficient teacher. One may even say that a faulty curriculum carried out by scholarly and inspiring instructors would probably produce a better teacher than an ideal program administered by instructors who had no interest in their task and who did their work in a listless and perfunctory way.

The professional program for prospective teachers, then, is something more than its curriculum offerings or requirements. It includes the life of the school itself, which should provide many opportunities for informal contacts between instructors and students, and also the teaching which goes on in the classrooms. Enthusiasm for one's work and devotion to the interests of the learner are qualities of the artist teacher for which there are no substitutes—and these qualities are not taught to the novice, they are rather caught from his instructors if his instructors exemplify them in a striking way.

The ideal preparation of the teacher, then, would include the privilege of working with such instructors.

Reading in Mathematics

By VIRGINIA WATTAWA
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RECENT STUDIES show that perhaps about twenty per cent of the failures in ninth year algebra may be attributed to the inability of the students to read their problems so as to understand them. What is not so commonly recognized is that the reading of all mathematics is not only harder than the reading involved in the study of non-mathematical subjects, but perhaps requires a special technique which has not been developed. Consequently children experience more difficulty in reading mathematics than teachers realize, for neither in the English classes nor in the mathematics classes does the average child receive adequate training in this type of reading.

In an attempt to prove that mathematics reading is difficult, Thorndike induced a group of Columbia students to participate in an experiment which consisted in reading a passage from an engineering book and then applying the information gained to a set of problems. None of the students who took part in this experiment were registered in mathematics or engineering classes. The adults who took part in this investigation reacted in much the same way as do children confronted by a similar situation in elementary mathematics. They floundered around and, as if in search of almost any way out of their predicament, made various incoherent attempts to obtain the solution. Not one of the solutions presented was correct.

A partial explanation of such a situation as just described may be that mathematics text-books presuppose some previous knowledge on the part of the reader, and so do not always state in detail the information required for an understanding of a definite paragraph. What seems simple to the author is sometimes far from obvious to the reader. Phrases like these: it is readily verified, it is so easily seen, it is geometrically obvious, abound in both our college and elementary text-books. It is more or less necessary to read between the lines. An incident illustrative of this occurred last year. A college girl of junior rank, specializing in English, was granted permission to enter a mathematics class without taking the

course required as a prerequisite. The exception was made because she was an unusual student. She ranked high in the first quarter of her class. Rather early in the course, problems were discussed whose solution was dependent on the theorem that if two lines are parallel they have the same slope, but if they are perpendicular, the slope of one is the negative reciprocal of the other. The girl, unfamiliar with this statement, was not content to accept it. She asked for a book which would give the proof, feeling assured that she could follow it. She studied the paragraph and then came back to request help. As is more or less usual, the text omitted the reasons for some of the statements. This simple proof made the assumption that the student knew about coordinate axes, knew that the slope of a line is the tangent of the angle which the line makes with the X axis, that the tangent is the ratio of the Y distance of a point on the line to the X distance, and that $\tan \alpha = -\tan (90-\alpha)$. The trigonometric formula proved to be the stumbling block in this particular instance.

A similar situation exists in high-school mathematics. Much of a pupil's success depends upon his retention of mathematical facts supposedly learned in the grades, and his acquisition of a mathematical vocabulary. There is no doubt that the learning of first year algebra would be made easier for students by the omission of unnecessary technical, semi-technical, and non-technical terms. In an examination of the first 50 pages of three standard texts, Thorndike found 50 words of unusual difficulty. Ten of these words follow: potentia, aggregation, resultant, haw, scalepan, Oughtred, projectile, papyrus, displacement, and Herigone. In an examination of two carefully written text-books, Thorndike found 580 words not among the first ten thousand in the language. However, this does not mean that new words should be omitted from the course, but that words chosen should be like axes, appropriate, assumption, words which will be of immediate value to the student.

It is fairly familiar to every teacher of mathematics that the statement of problems in the text is sometimes ambiguous. The following illustration is taken from a mathematics of finance text. "A father begins when his son is 7 years old to lay aside $400 at the end of each year for his son's college education. If all sums can be accumulated at 5% until the fund is exhausted, how much will be available at the beginning of each of the four years of the boy's college course, if he starts to college at the age of 17?" A possible

interpretation of this problem is that the father puts aside $400 each year until the son is a college senior. The desired interpretation is that the father puts aside $400 for ten years. Another example taken from an advanced algebra text, shows this inexactness of statement. "A's age is double B's. Six years ago A's age was three times B's. How old is each?" There is frequently failure on the part of students when solving problems of this type, to recognize the fact that A's age six years ago is to be compared with B's age six years ago.

The reading difficulties which the poorer mathematics students encounter fall into two main classifications, failure to comprehend the conditions of the problem or understanding the problem, failure to read the numerals correctly. Psychologists admit that the reading of numerals makes unusual demands upon the reader, and offer as a possible explanation that the combinations of numerals in digits are different, while the combinations of letters in words remain stable. Statistical clerks commonly use a straightedge to indicate the line they are following across a page, when they copy or read numerals. Even a poor prose reader does not need to do this. It is also known that the reading of numerals combined with words involves a different procedure than the reading of isolated numerals. Investigation has shown that the numerals of a problem make greater demands on a reader than do the accompanying words. Experimentation with adults has shown that they ordinarily read and re-read a problem. The first reading is for fixing in their minds the conditions necessary for the solution of the problem, and the second reading is to perceive the numerals. Two ranges of perception are noticed. Shorter numerals and familiar ones receive whole first readings while longer numerals receive partial readings. However all subjects re-read the numerals when they begin the solution of the problems. Evidently children should be taught to read in this fashion. However, when some children read problems, there is a tendency to neglect all but the figures involved, and they do not read and re-read. They should be trained to neglect the quantitative elements until they comprehend the meaning of the words. There is a recent movement to teach grade children reading in this manner. It might be well to adopt this idea in high-school classes, if there are pupils who have difficulty in understanding problems.

The following is a suggestion for teaching the comprehension of

problem reading to ninth grade pupils. The history of the world flight is told briefly thus:[1]

Distance: 27,534 miles
Flying time: 14 days, 15 hours, 11 minutes
Speed: 76–36 miles an hour
Start: Seattle, Washington, April 6 Sunday
End: Seattle, Washington, Sept. 24 Sunday
Average distance of hop: 483 miles
Gasoline used: 21,060 gallons
Spanned: 21 countries, 21 states, one territory
Four world records established

Then the pupils might be given the following short test in which they are asked to choose the ending which would make a true sentence.

1. The average speed was (a) 75 miles an hour
   (b) 175 miles an hour
2. The time they were actually flying was about (a) 6 months
   (b) 1 month
   (c) ½ month
3. The distance traveled was about (a) 7000 miles
   (b) 27000 miles
   (c) 20000 miles
4. The mileage obtained by each plane was (a) 4 miles per gallon
   (b) 3 miles per gallon
   (c) not given

Probably data on more recent flights would arouse more interest.

Possibly if silent reading were a more common practise in high-school classes, pupils would receive better training in comprehending their problems. The customary procedure has been to read a problem in a rather casual fashion; teachers too perhaps have neglected the opportunities which the explanatory pages of the text offer for silent reading.

In a comparatively new text-book,[2] the author suggests a scheme for drill in silent reading of arithmetic problems; the idea might possibly be of use in ninth year algebra classes. He suggests having cards, on each of which a different problem is written. These cards are to be distributed among the members of the class. Each child reads his problem silently, then tells it in his own words to the other members of the class, and then all attempt to solve the problem from the re-statement.

The problems contained in the standard algebra text-books are the target of much criticism, the charge is made that many of these

[1] Ch. V. p. 131, 24th Yr. Bk., Nat'l Society for the Study of Education.
[2] Lundquist: *Modern Arithmetic.*

problems are of little value in the situations of real life. In a thesis submitted for a doctor's degree at Columbia, Benjamin Daily related an experiment conducted in the classes of some of the New York schools, which attempted to meet this criticism. He formulated a number of test problems for ninth-year students. Some of these problems did not contain sufficient data for the solution, others gave too many facts; still others were the conventional type of problem. The hypothesis on which this experiment was conducted was that if the pupils are taught to select data necessary for the solution of mathematical problems, they will improve in the ability to select data in other fields of learning and in the problems of real life. The results of these tests showed that relatively few pupils could successfully solve problems where insufficient or unnecessary data was introduced. They seemed to be unable to sift the facts which they needed from the mass of irrelevant data. This was especially true when the pupils were not aware of the character of the problems.

It might be possible to introduce into the algebra classes problems which in themselves would be more interesting. Of what value is a problem like this, "One man is twice as old as another. The sum of their ages 12 years ago was $\frac{1}{2}$ the sum of their ages 12 years hence. How old are they?" Where except in an algebra class would such a situation as stated in this problem exist? Surely at the present time when our schools are being subjected to the closest kind of scrutiny we can no longer find justification for material taught in our mathematics classes which has little or no value. There has been a decided change in the material presented in some of the newer arithmetic text-books. A recent series of arithmetic texts[3] has drawn material for its problems from every phase of modern life. In these books are included problems about our state and national governments, our business organizations, great industries, and transportation. There are problems which teach the metric system and the inter-relationship of mathematics and science. Even facts from the lives of our great men are skillfully woven into problems. The introduction of material of this sort into the teaching of algebra would surely be a rather radical departure from the traditional program. However, it might win the favor of critics who feel that the subject has little to commend it. More important than this it might really make the teaching of algebra more vital.

[3] Smith, Luse, Morss: *Problem and Practice Arithmetic.*

If a teacher were interested in studying the reading disabilities of an algebra class, the first thing to do would be to keep a record of the particular words or paragraphs that the class has difficulty in understanding. Then a record might be kept of the problems the class likes and of those in which they manifest little interest. It is only by proving in what respects the present course is poor, that a newer more constructive program can be organized. The teacher might devote class time to silent reading as well as to oral reading, and perhaps might make some test of the ability of the students to read algebraic problems before and after the trial. Finally, by omitting topics of no vital importance like the finding of the square roots of algebraic polynomials, work problems, river problems, age problems, she might find time to experiment with some of the newer problems of the type mentioned.

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Supplementary Projects: Purpose

By J. S. GEORGES
Crane Junior College, Chicago, Illinois

ONE OF THE outcomes of individualized instruction is enabling the pupil to proceed at his own rate and to complete the unit as an individual task rather than a group task. Individual differences between the rates of learning as well as rates of application often produce a state of affairs in the teaching program that discourages the new teacher using the individualized method of instruction. The bright pupil may complete a unit or an assignment a day, two days, or even a week before the rest of the class. The question arises as to what to do with the bright pupil or the pupil who learns with facility and works rapidly.

Supplementary project work is a device to take care of this difficult situation which may arise in connection with individualized instruction.

Again, there are certain phases of a unit of instruction, certain manifestations of the central concept or process, certain important applications to scientific phenomena, and certain historical backgrounds that cannot very well be taken up in the classroom. These phases of the unit are however significant. They contribute definitely to a meaningful interpretation of the unit. For example, the theorem of Pythagorus may be a unit element in a unit in geometry or a unit in itself in a course in general mathematics in the junior high school. In connection with this important theorem, there are certain interpretations that cannot be taken up in the classroom work. They are interesting, helpful, and illuminating, but yet they are not regular classroom work. If the demonstration of the theorem is a part of the unit, clearly it is out of the question to present more than one or at most two different proofs. Yet there are over 100 proofs of this important theorem, some of them important historically, some of them interesting in nature and application, and others illuminating because of their association with great mathematicians. It would add to the appreciation of the importance, significance, and nature of the theorem of Pythagorus if some of these demonstrations were presented to the class, perhaps as a special report either by the teacher or by the pupil.

Supplementary project work aids in thus supplementing regular

classroom work. It is a means of bringing to the attention of the class important historical developments in connection with mathematical ideas, principles, or theories. It is a means of increasing appreciation for the power of mathematics as a science by bringing to the attention of the class specific contributions of the science to the development and progress of civilization. It is a means of convincing those pupils who are reluctant to find any interest and joy in the study of mathematics that the processes, concepts, and ideas of mathematics are standards for similar activities, thoughts, and ideas in non-mathematical fields. These special reports are more interesting to the class if made not by the instructor, but by the pupil, for if they are made by the instructor, they may be considered and are considered by the class as not being different from regular classroom work; while if they are presented by a member of the class, the reports have a new significance and new appeal to the individual as well as to the group.

Supplementary project work then is a device for enlarging the scope of instruction, for broadening the horizon, for expanding the activities; and in this extension of the meaning and scope of the unit, the pupil takes the major responsibility.

Perhaps one of the best uses to which supplementary projects may be put is in the handling of exceptional cases. Just as the bright child offers a problem in individualized instruction, even so the exceptional mind. By the exceptional mind we mean those capable pupils who have a special dislike for mathematics. In a sense they are not problem cases for they are not mentally deficient. They do satisfactory work in most if not all of their other courses, except in mathematics. If the difficulty is not one of mentality, then surely it is one of interest. For example, consider the case of a girl who is specializing in art. She is an average pupil in all of her classes except in art and in mathematics; in art she is exceptionally brilliant, she likes the work, she has inclinations and capacities for expressing herself, and she does it. In mathematics she has great difficulty; she is not interested, does not apply herself, and does not care whether she learns or not. Supplementary project work offers a solution to the problem of this specific girl. Her instructor in mathematics noticing her special aptitude and inclination toward art indirectly arouses her interest in mathematics through art. She is willing to work on a project whether in mathematics or not, provided she can draw. A supplementary project based on

harmony of line and color or one based on static or dynamic symmetry may give this pupil a new viewpoint of what mathematics is, and of what mathematics may mean to those who have an understanding of its concepts.

Again, take the case of a pupil who is interested in languages but not in science. He has to take two years of mathematics as required work, and though he does exceptional work say in French or Latin, his work is below credit level in mathematics. In this case also the supplementary project may be the means of arousing the interest of the pupil to find mathematics a language in the sense that French or Latin is a language. Some manifestation of algebraic language, such as algebraic function or transformation or even the development of algebraic symbolism may be assigned as a project. To the pupil who is primarily interested in a language as a mode of expression, algebra has a unique appeal, because it also is a language, a language of quantities, of changes and of variations.

What the classroom often fails to accomplish, the supplementary project work does; namely it solves the problem of the exceptional pupil.

Finally, supplementary projects may be used by the teacher of mathematics for the purpose of inspiring those pupils who have a special aptitude for mathematics and have a special liking for the subject. The pupil who has a capacity for keen mathematical analysis or takes easily to mathematical and logical reasoning needs an intelligent guidance. It is the duty of the teacher of mathematics to inspire such pupils toward higher attainment in the subject. The ranks of mathematicians must be filled by recruits from secondary schools, and the best method of recruiting young mathematicians is to bring them face to face with the great masters of the science. Supplementary projects enable the mathematically inclined pupil to see the glorious country that he himself may explore some day. Used discreetly and understandingly, supplementary projects in mathematics may be used as a guiding program, discovering early the pupils who have a special interest in mathematics and inspiring and guiding them on from unit to unit and from course to course until they answer the call of the muse of mathematics.

A Play, "The Case of Matthew Mattix"

By Alice K. Smith
Tallahassee, Florida

Cast of Characters

Ida Wanna Work, Hayta Work, Wida Wake } School Girls
Aunt Něva Work, a lady of leisure.
Ecks, a mysterious character.
Bella Dumm, a rather bright girl.
Newsboy, heard but not seen.
Mr. G. Otto Work, a burdened parent.
I. Wood Work, never idle.
Hunter Work, not so industrious.
Archie Teck, always planning houses.
Abel Bilder, a contractor.
Mīnda Range, an *X*-cellent cook.
Miss Nita Needles, who sews a fine seam.
Miss Stattie Stitian, a sociologist.
N. A. Burr, who lives nearby.
Al Gebra, very fond of Ecks.
Non-speaking parts:
Matthew Mattix, the arch-criminal.
G. O. Mettry, accomplice of Matthew Mattix and Al Gebra.
A surveyor, a manufacturer, a bridge builder, a navigator, and an astronomer.

Scenes

Scene I. Living-room of the Work home. It is late in the afternoon of a spring day of the early 1930's.
Scene II. The same. Two weeks later.

Scence I

Ida Wanna (*Throwing down pencil in disgust*). I can't do this problem. Listen to this. (*Reads*) "A dog on the Manakin Road barks 40 minutes every time anyone passes by. How many passers-by will it take to keep him barking all night, allowing $10\frac{2}{3}$ hours?"

Hayta. That's nothing, absolutely nothing. Can you feature this? (*Reads*) "A mouse is at the top of a poplar tree 60 feet high, and a cat is on the ground at the foot of the tree. The mouse descends $\frac{1}{2}$ of a foot each day and at night it turns back $\frac{1}{6}$ of a foot. The cat climbs 1 foot a day and goes back $\frac{1}{4}$ of a foot each night. The tree grows between the cat and the mouse $\frac{1}{4}$ of a foot each day, and it shrinks $\frac{1}{8}$ of a foot every night. In how many days will the cat reach the mouse, and how many feet has the tree grown in the meantime, and how far does the cat climb?"

(*A slight pause during which the girls look at each other in disgust. Aunt Neva yawns*)

WIDA WAKE (*Entering R. hurriedly, much excited*). Oh-Oh-Oh-I'm *so* scared! I was almost home when I saw that terrible Matthew Mattix coming toward me, with some of his friends.

HAYTA WORK. Not Al Gebra?

WIDA. Yes, Al Gebra and that frightfully plain G. O. Mettry!

HAYTA. What a narrow escape! (*Noise is heard outside.* HAYATA *goes to window.*) Oh, look out of the window!

WIDA (*Going to window*). Omigoodness! There's the police wagon! Oh, they're arresting Matthew and Al and G. O! They've got them in the wagon!

HAYTA AND BELLA. Goody! Oh, boy! (*Go to table, close books, throw them down, pick up comic papers and start to read, giggling*)

(*A knock is heard at left.*)

AUNT. Who is that little boy in the back yard? (*Goes R. back of curtain.*) Come in, little boy.

(*Enter Ecks, L., half sobbing*)

Who are you?

ECKS. I'm Ecks.

WIDA. Oh yes, the great unknown! He looks rather tiny to me, though.

AUNT. Are you lost?

ECKS. Yes'm, I am, and I know my father is looking for me.

AUNT. Who is your father?

ECKS. Al Gebra.

AUNT. Oh yes, I've heard that he's always losing Ecks and getting everybody to help him find him.

BELLA DUMM. I've tried to find Ecks lots of times, but I never could. Sometimes I'd think I had him, but he always turned out to be a minus quantity.

IDA WANNA. I'm positive I could have found him, but it was always too much trouble. And now that we have found him, will some one please tell me what we are going to do with him?

ECKS. Take me to my father!

HAYTA. We can't do that. He's been arrested and taken to jail. He's such a low cuss that I should think you 'd never want to see him again.

ECKS. Oh-h-h-h! (*Aunt puts arm about his shoulders.*)

AUNT. Hayta, I'm really ashamed of you.

(*Curtain*)

SCENE II

(*Girls and Aunt seated. Loud crash heard. Girls jump up.*)

WIDA. What was that?

BELLA. Some one must have dropped a perpendicular.

NEWSBOY. (*Outside*) Wuxtra! Wuxtra! Wuxtra!

(*Continues calling until Hayta comes out. His voice is heard after that, more and more faintly as he apparently goes down the street and out of hearing.*)

HAYTA. Oh, there's a newsboy with an extra!

AUNT. Take this nickel and buy one. (*Hayta takes nickel and goes out. She is heard asking the boy for a paper, and returns immediately.*)

HAYTA. Oh! (*Reads*) Matthew Mattix, Al Gebra, and G. O. Mettry have been sentenced to death! (*Aunt takes paper.*)

AUNT. I wonder what the charges were. (*Hayta hands paper to Aunt.*) (*Reads*) For cruelty to dumb animals, arson, grand larceny, mayhem, and homicide!

WIDA WAKE. Auntie! Not really!

AUNT. It's true, though. They cruelly tore Don Roucks and Ronald Sloyd off the football team, caused Hester Cholley to burn up the midnight electricity, stole so much time from play that Jack became a dull boy, cut George Apthorpe down 30 pounds, made a number of pupils study their arms off, and killed thousands by overwork!

ALL. Oh-Oh-Oh-Oh! How horrible!

HAYTA. But at last they'll be put out of the way.

IDA WANNA. Glorious! Now I can spend all my study periods reading *College Humor* and *Ballyhoo!*

BELLA. Me, too. But I'm sorry for them. Their sorrow must be acute.

IDA WANNA. Don't be obtuse!

BELLA. Think how you and I would feel.

IDA WANNA. That's not a parallel case. Any way, we'll never meet them again, however far the sentence is extended.

HAYTA. I certainly hope they'll keep at an infinite distance from me!

(*Enter father, Mr. G. Otto Work. Sits in chair, and groans. Girls gather around him.*)

HAYTA. Father, what *has* happened?

(*Mr. Work groans.*)

AUNT. Are you hungry, Brother? Here, eat this pie I saved for you.

(*Hands him piece of pie, made from card-board, on which the letter π is plainly visible.*)

MR. WORK. No, Sister, the root of my trouble lies deeper than that.

IDA WANNA. Well, can't you have it extracted?

MR. WORK. (*Shakes head and groans.*) Children, there is a radical change in our affairs. I have lost my job in the controller's office.

ALL. How? Why?

MR. WORK. You see, they are putting Matthew Mattix out of existence, and we can do nothing. Everyone in the controller's office, the treasurer and his force, the road department, in fact, every one except the janitor is out of work. And since even a broom handle cannot be made without the aid of Matthew Mattix, he'll soon be wandering the streets, too.

(I. WOOD WORK *enters, carrying carpenter's tools*)

I. WOOD WORK. (*Breathlessly*) Have you heard what has happened?

WIDA } What now?
HAYTA } Anything more?

I. WOOD. Everything! Our saws wouldn't cut straight, our squares turned to rhombuses, and when we tried to lay some brick, everyone of them turned into oblique parallepipeds! Telephones are out of order, the water works have broken down, and not an automobile will run because all the cylinders are missing. They can't weigh things in the grocery stores, can't make change, and the dry goods stores have closed because they cannot measure. Everything is in the wildest confusion! The telegraph has ceased communication, and we are cut off from the rest of the world. Just now an aeroplane fell to the ground, probably because the aviator could not find his altitude and angle of elevation!

BELLA. But how did it all happen?

MR. WORK. Don't you understand? Science depends upon Matt-

new Mattix for it's very existence. Naturally, without Matthew Mattix the works of Science must cease.

(HUNTER WOOD *Enters, carrying gun*)

BELLA. Hello, did you get anything?

HUNTER. No. Couldn't hit a thing. Every shot, instead of going in a decent parabola, went zigzagging about at random, and I had to stop for fear one would fly off at a tangent and kill a cow.

BELLA. Gosh. I didn't know there was any need of Matthew Mattix in the course of a bullet.

MR. WORK. Don't show your ignorance, Bella. Have you never heard of Triggernometry?

(*Architect, surveyor, builder, manufacturer, bridge-builder, navigator, astronomer, and others ad. lib. pass across stage, gloomy and dejected. They pile up their instruments in the corner.*)

ARCHITECT. These things can all go into the trash can now. We've no further use for them. Eventually we'll all go back to living in tepees.

BUILDERS. Nothing so good as tepees! Do you think for a minute that we could erect a vertical pole without the aid of Matthew Mattix?

ARCHITECT. Right, Mr. Builder. (*Puts his head in his hands.*)

MR. WORK. There's nothing left for us to do but to sit down and wait for the end. Even then we won't be able to erect a perpendicular to ascend to Heaven on.

IDA WANNA. Let's turn on the radio.

MR. BILDER. Don't you know that the radio depends upon the most intricate calculations of Matthew Mattix? He even has to call in his friends, Cal and Anna, to help.

HAYTA. Cal who? Anna who?

MR. BILDER. Cal Culus, of course, and Anna Lytics.

(*Enter* COOK *L.*)

COOK. Please, Miss, It's just terrible. Nothing I've cooked has come out right. The bread won't rise, the jelly won't jell, and the cakes have fallen!

MR. WORK. Matthew Mattix needed again! You simply couldn't get the right ratio.

COOK. Yassir! (*Exit* COOK *L.*)

(*Enter* MISS NITA NEEDLES *R.*)

MISS NEEDLES. Madame, I can't do anything with this dress!

The sleeves, waist, and skirt are all out of proportionl (*Everyone shakes his head and looks gloomily at everyone else*) And the corresponding parts simply can't be made to coincide! Look at this! (*Holds up dress with sleeves ridiculously narrow and long, waist wide and short, skirt very tiny, much shorter and narrower than waist.*)

AUNT. Well, I certainly cannot wear that. (NITA *takes dress. Exit Nita.*)

(*Enter* STATTIE STISHAN)

STATTIE. Hello, folks. My job is gone, like all the rest. All the statistics owed their existence to Matthew Mattix. We can't do much without them, so I'm not needed any more.

(*Loud noise heard outside.*)

(*Neighbor rushes in.*)

N. A. BURR. Hooray! Hooray!

ALL. What is it?

N. A. BURR. The electricians refused to electrocute their friends Matthew Mattix, Al Gebra, and G. O. Mettry. The governor has stayed the execution, and Matthew Mattix and his pals are set free! I brought them along with me.

(*Enter* MATTHEW, MATTIX, *etc.*)

MR. WORK. Friends, we welcome you here!

WIDA. I see where I've been making a sad mistake. It has been demonstrated to me beyond the shadow of a doubt that Matthew Mattix and Al Gebra are about the most useful people I know, and G. O. Metry will always look beautiful to me, and no longer plain.

N. A. BURR. You are quite right. Without them Science would be helpless. Construction would cease, and we would lose most of our luxuries and comforts. There would not even be any railroads or steam ships.

MR. WORK. We have been saved from a great calamity.

AL GEBRA. (*Weeping*) But all this means nothing to me. There will be no joy for me until Ecks is found.

(ECKS *rushes in and throws himself into the arms of* AL GEBRA)

ECKS. Daddy!

Join the National Council of Teachers of Mathematics!

The Slide Rule in the Junior College*

By W. W. GORSLINE
Crane Junior College, Chicago, Illinois

THE PROGRAM of this meeting was composed of three papers on the following subjects: The Slide Rule in the Junior High School, The Slide Rule in the Senior High School, and The Slide Rule in the Junior College. The first paper advocated that the slide rule should be studied in the seventh, eighth and ninth grades. No explanation of the foundation principles of logarithms was mentioned, and only the simplest operations of multiplication, division, proportion, squares and square roots were explained. If the student starts to study an instrument when he is young, even though he doesn't understand all the reasons, he will be ready for the further study of the foundation principles upon which the slide rule is based when he gets to the senior high school and junior college. When the pupil begins the study of logarithms in the third year of high school then he will be ready to understand fully all the reasons for any procedure with the slide rule.

When the pupil enters the college he will be ready to solve the many wonderful problems which he encounters in chemistry, physics, and the many other studies which he will pursue. I advocate very strongly that each college set aside one hour each week for the exclusive study of the slide rule. At Crane Junior College the author has taught four of these classes reciting once a week. A suggested list of lessons based upon such a course of twenty weeks will be given. For many years I have given these one hour classes to the engineering students only. The commerce and administration department urged that a class be devoted to commerce students only. For the past two years such a class has been given and this proved so popular that should a number of other classes be made available to these students, they would all be filled. All four of these classes have had a good course in logarithms and are competent to take up a thorough study of the slide rule and obtain a great deal of practice which is so vital to all, when undertaking the mastery of an instrument of this sort.

* Paper delivered before the Men's Mathematics Club of Chicago, December 16, 1932.

Lesson 1—Reading of the Slide Rule

A drawing of the slide rule D scale is given and the divisions are enlarged so the pupil will have no difficulties in seeing all the small divisions which are so bewildering to the beginner. At frequent intervals along the scales are small rectangles, in which the pupil is expected to place the exact reading indicated by an arrow to the division. Such a lesson will make sure that the pupil can read a scale of this kind accurately. On the same sheet an A scale is also enlarged and various readings are asked of the pupil.

Lesson 2—Multiplication Using C and D Scales

An introduction showing how the slide rule was invented and how these scales were constructed. Cadjori "The Logarithmic Slide Rule" is made use of to give the historical background. Numerous problems in multiplication, using number relations, as well as a few problems in business. Such a lesson sheet is perforated and may be torn out and handed in to the instructor for correction.

Lesson 3—Division Using C and D Scales

Numerous diagrams illustrating how division is carried out and a problem sheet with many applications from engineering and business.

Lesson 4—Proportion

Simple and continued proportion solved by using only one setting of the scales. Use is made of the constants on the back of the rule.

Lesson 5—Combined Multiplication and Division

The location of the decimal point is usually given by inspection and in a late pamphlet on the subject, a problem is given and the point is placed by this method. The answer given is wrong. This method is excellent for problems where one is sure of the magnitude of the answer. If a number is close to 10 or some multiple of ten the operator may estimate it as nine when it turns out to be a number greater than ten and thus the answer will be wrong.

The method which I use makes use of the logarithmic characteristics. Some rules which have been devised made use of the projection of the C scale and the number of significant places in the number. If the C scale projects to the right subtract one for multiplication and add one for division. When one tries to use the rule for other scales such as the A and B or folded scales, difficulties arise at once.

The best way is to cover the entire subject of placing the decimal

point by the use of one rule which will apply to all scales. The rule may be illustrated by the use of the following problem:

$$\begin{array}{cccccc} 1 & -2(+1) & 1 & 0 & & \\ 46.3 & \times .0543 & \times 23 & \times 1.32 & & \\ \hline 698 & \times 1254 & \times .0025 & \times .7 & & \end{array} = .0497$$
$$2(+1) \quad 3 \quad -3 \quad -1 \quad +1$$

Write the characteristics above the factors in the numerator and below the factors in the denominator.

Set the hair line above 463 on the D scale.

Move the slide so that 698 falls below the hair line.

The C scale will extend to the left and the left index of the D scale will be between the ends of the number 698 and this number is said to bridge the index. Whenever this happens, +1 is added to the characteristic of the number.

Move the hair line to 543 on the C scale. Bridging takes place here also so +1 is added.

Move C scale so 1254 is below the hair line.

Continue this process until the problem is fully worked.

Adding the characteristics in the numerator and subtracting those in th denominator, we get −2 which is the characteristic of the answer.

This method has never occurred in print before and is unique.

In this lesson many applications from chemistry and the sciences can be worked out.

Boyles' law and Charles' law give some excellent applications.

Lesson 6—The C. 1. Scale

The quadratic equation is solved with ease.

Lesson 7—The Folded Scales

Lesson 8—Squares, Cubes, Square Roots, Cube Roots, by Using the A, B, C, and D Scales

Lesson 9—Raising a Number to Any Power or Extracting Any Root

Lesson 10—Solving the Cubic Equation by Only One Setting of the A, B, C, and D Scales

Any cubic equation of the form $x^3+Bx^2+D=0$ can be solved at once. There is no comparison in the time spent with Horner's method which requires considerable work, as all mathematicians are fully aware. Even though a first degree term in x does appear it can be eliminated with ease. So far as I know no attempt has ever been made to solve cubics in this manner.

Lesson 11—Miscellaneous Exercises covering use of all the scales and a review of all work previously learned. Many problems are given for drill.

Lesson 12—Sines of All Angles no Matter How Small or How Close to 90°

Use is made ot the two gauge marks on the sine scale, namely; the second's mark and the minute mark. Sines of angles, as small as a thousandth of a second, and sines of angles like 89°59′59.925″ can be worked out with ease.

Lesson 13—Solution of Right Triangles Containing Small as well as Large Angles

Lesson 14—Solution of Oblique Triangles Using Law of Sines

Lesson 15—Tangents of All Angles, Small as Well as Large

Lesson 16—Solution of Right Triangles Involving Tangents

When the sides differ by 1 the angles are found to seconds.

Lesson 17—Solution of Oblique Triangles, Using Law of Cosines, Law of Tangents and the Half Angle Formulas

Lesson 18—Miscellaneous Problems Involving Angles with Trigonometric Application

Lesson 19—Business Problems Involving Discounts, Simple and Chain, Percenages

Simple Interest, as well as Compound Interest

Lesson 20—A General Review of All Processes

A course of this kind fully prepares a student to pick up his slide rule at any time and solve his problem with speed and accuracy. Such a text book planned with the lesson idea is available now. Each lesson sheet is perforated so it can be torn out and handed in to the teacher. With such a thorough course as the one planned above the slide rule will cease to be a "guessing stick," as some teachers call it, and become a real educated stick which will be a life-long companion.

Don't fail to secure the Eighth Yearbook of the National Council of Teachers of Mathematics on "The Teaching of Mathematics in Secondary Schools." The supply is limited. Send $1.75 to The Bureau of Publications, Teachers College, Columbia University, 525 W. 120th Street, New York City, for a copy.

Does Mathematics Condition Human Progress?*

By U. G. MITCHELL
State University of Kansas, Lawrence, Kansas

Empirical Basis of Mathematics. The fact that mathematics originated in human experience leads us to wonder if mathematics, in its entirety, is not merely the abstract form of the world about us. Not the abstract form of the world of physical phenomena alone, to be sure, but of the logical and psychological phenomena of the intellectual world as well.

We recall that the ancient Greek philosophers, certainly entitled to rank among the great thinkers of all time, were able to express in terms of mathematics so many of the concepts and relations in the world about them that they came to believe that all creation was merely the carrying out of a mathematical plan preëxistent in the mind of the Creator. Their point of view is well expressed by Nicomachus of Gerasa, a Neopythagorean who flourished about 100 A.D. and is the best known of early Greek writers on arithmetic:[1]

> All that has by nature with systematic method been arranged in the universe seems both in part and as a whole to have been determined and ordered in accordance with number, by the forethought and the mind of him that created all things; for the pattern was fixed, like a preliminary sketch, by the domination of number preëxistent in the mind of the world-creating God, number conceptual only and immaterial in every way, but at the same time the true and the eternal essence, so that with reference to it, as to an artistic plan, should be created all these things, time, motion, the heavens, the stars, all sorts of revolutions.

Few modern mathematicians would be likely to accept the mysticism of the Pythagoreans; but it is probably safe to venture that nearly all of them would agree with the following paragraph, not greatly different in essential thought, written by one of the best known mathematicians of our time:[2]

* This paper formed part of an address given at the Mathematics Sections of the Kansas State Teachers Association in Topeka, November 5, and the Missouri State Teachers Association in Kansas City, November 11, 1932.

[1] *Introduction to Arithmetic*, Book I, Chapter VI. English translation by D'Ooge, University of Michigan Studies, Vol. XV (1926), p. 189.

[2] A. N. Whitehead, *The Organization of Thought*, London, 1917, pp. 14–15.

Algebra is the intellectual instrument which has been created for rendering clear the quantitative aspects of the world. There is no getting out of it. Through and through the world is infected with quantity. To talk sense is to talk quantities. It is no use saying the nation is large—How large? It is no use saying that radium is scarce—How scarce? You cannot evade quantity. You may fly to poetry and music, and quantity and number will face you in your rhythms and your octaves.

Perhaps the most surprising thing about mathematics is that man invented it to express, in a precise and compact manner, ideas drawn from observation and experience, and later found that he was able by logical processes to formulate other ideas independent of further observation and experience. Mathematics proved to be cumulative, and its rate of increase has continued to grow until more than five hundred journals are now devoted in part and more than twenty exclusively to its publication. It is estimated that more than fifty thousand different books and memoirs have come from the mathematical press in the last generation.

But what, if anything, does all this vast accumulation of symbolism, diagrams and inferences represent? The mathematician, of course, long ago declared his independence of any concern in the matter. It is his business to produce more mathematics regardless of whether or not it has any use except that of intellectual satisfaction. His attitude seems to us entirely justifiable. There is no more reason why a mathematician should be called to account for developing new mathematics than there is for asking a musician what use there is in composing a symphony.

Mathematical Basis of Empiricism. If mathematics, resting originally upon an empirical basis, continues in its higher forms to represent abstractly concepts, processes and laws in our physical and intellectual environment, ought it not to happen, occasionally at least, that men versed in some particular phase of mathematics are enabled thereby to comprehend some physical or intellectual phenomenon not previously revealed by empiricism? In other words, if man through an understanding of concrete situations is led to their abstract mathematical forms, ought he not, sometimes, through an understanding of abstract mathematical forms, to be led to an understanding of concrete situations?

The answer is that it ought to happen and has so happened many times. One of the most familiar and oft-quoted examples is the fact that the ancient Greek mathematicians developed in great detail the properties of conic sections as a matter of pure intellec-

tual enjoyment. Through many centuries no one saw any particular use for their rigorous and elaborate theory until Kepler, familiar with their work, wondered if their beautiful ellipses might not be the paths of the planets. The Greeks had mapped the heavens and prepared a complete, ready-at-hand analysis of our solar system, but it was nearly 2,000 years after their time before any one knew it. A still more striking example of mathematics revealing a life-situation before experimentation had discovered it grew out of Kepler's work. After he had published his laws of planetary motion there was a vast amount of experimental work carried on to check his theories. It was found that one of the planets, Uranus, perceptibly deviated from the course predicted for it. In attempting to explain its perturbations, two astronomers, Adams in England and Le Verrier in France, working independently, assumed the influence of an unknown planet and, in a series of remarkable computations, estimated its size and position. Le Verrier communicated his results to Dr. Galle, an astronomer in Berlin who possessed charts of the known stars and planets. Dr. Galle directed his telescope toward the position indicated and in less than 24 hours after the receipt of Le Verrier's letter had discovered the planet Neptune. A planet more than 60 times as great in volume as our earth had traveled its unending journey about the sun unnoticed and unnamed until mathematicians found it without seeing it.

Other examples come readily to mind. Chemists have discovered new elements because gaps in the tables of atomic numbers indicated that such elements should exist. Mathematicians pointed out to physicists the existence of conical refraction and to geologists the limitation of possible types of crystals to 32 before these facts were discovered in the laboratories. More interesting to us than any of these is the fact that in our own time world-famous scientists have been busy for 20 years trying to determine by experimentation whether or not the revolutionary theories of Einstein, developed from higher mathematics, explain correctly the riddle of the physical universe. It is needless, however, to multiply examples. Every mathematician has enjoyed, at some time or other, the pleasant experience of having his mathematics illuminate with sudden clarity something already familiar but not comprehended.

Mathematics and Human Progress. The past century has witnessed astonishing progress in the development of pure mathematics. It has also witnessed a marvelous development in science

and invention. There may be more connection between these two things than at first thought seems probable. Shaw tells us that the existence of the wireless telegraph is due to deductions of Maxwell by means of theorems that depend upon the square root of minus one and that the possibility of the long-distance telephone depended upon investigations of Pupin by means of theorems that depend more directly upon the modern theory of expansions in fundamental functions.[3] If these statements are accurate, it would appear that these inventions could not have been made in the way in which they were made without the development of the necessary types of mathematics. As Professor Crathorne pithily remarks, "And men go up in flying machines made of wood, wire, complex fractions, gasoline, and exponents."[4]

Modern science attempts to subject its investigations to mathematical treatment. Consequently study in any field becomes scientific as fast and in so far as it becomes mathematical. What law of physics, chemistry or astronomy, the most scientific of modern disciplines, is not stated precisely in terms of mathematics? Research workers in education, biology, psychology, medicine and sociology see increasingly ahead of them a need for higher mathematics. The educated reading public is even becoming aware of the mathematical basis of modern science. The following paragraphs, appearing under the caption "The Key of the Universe," seem to the writer surprizing statements to be found in the editorial columns of a great and popular weekly:[5]

> Mathematics, if we are not greatly mistaken, is presently destined to play a much larger part in our general scheme of education than it ever has in the past. One is forced to this conclusion not by the insistent demands of students but by the consideration that the tools and the methods offered by this science have been so largely responsible for the extraordinary advances in other sciences which the past generation has witnessed. The more mathematics contributes to the development of other sciences the more dependent upon it they become.
>
> This state of affairs has come to such a pass that already the layman who wishes to keep up with modern thought is restricted to the use of the most elementary books, for if he goes to the more advanced works, the sources of really sound and substantial information, he at once finds his progress barred by an entanglement of calculus and other branches of higher mathematics which he either has not studied or has completely forgotten.

[3] J. B. Shaw, *Lectures on the Philosophy of Mathematics*, Chicago, 1918, p. 1.

[4] A. R. Crathorne, "Algebra from the Utilitarian Standpoint," *School Science and Mathematics*, Vol. XVI, p. 431.

[5] *The Saturday Evening Post*, issue of June 1, 1929, p. 30.

In electricity, physics, chemistry and astronomy the educated reader might expect to encounter this difficulty, but he may be surprized to find that he will be no better off if he attacks biology, physiology or any serious discussion of the structure of matter, of the nature of the atom, or of modern conceptions of time, space and what we call the universe.

It may be possible that much of our accumulated mathematics will never be found to represent anything known to us in the world around us. It may also be possible that much of it which would be extremely useful if humanity could but see what it represents will remain uninterpreted for ten thousand years. It may even be true that the theory of mathematics is so far ahead of its applications that all the mathematics needed by scientists for the next thousand years is already in print; but it would certainly be unwise to cease research work in mathematics on any such supposition, for we cannot by any means be certain that the kind of mathematics already developed is the kind most needed by civilization.

The fields of psychology, education, sociology and government are old as phases of civilization but new as fields of scientific study. The fact that the problems in these fields have not yielded so readily to mathematical treatment as those in the natural sciences may be due to a failure on the part of mathematicians to produce the necessary types of mathematics or to a failure on the part of the workers in these fields to be sufficiently trained in mathematics; but the suggestion comes persistently to mind that the misgovernment, social maladjustments and general human unhappiness and unrest so prevalent in the world today are due to the fact that the scientific method with its mathematical precision, order, arrangement and exactly stated laws is only beginning to penetrate these human-relations fields.

Great masses of data, for example, pertaining to commerce, marketing, banking, trade-restrictions, business-cycles and other economic conditions and relations have been collected and published; but the experts in these fields seem to be unable to marshal the variables involved into such precise forms and orderly arrangement that their relations to each other and their general significance as a whole can be grasped and understood sufficiently to avoid economic maladjustment and distress of the severest kind. There seems to be a need for the condensing power of some such symbolism as is employed in mathematics, if not that of mathematics itself. The mathematician can think of a thousand million

simultaneous equations in a thousand million variables merely by writing

$$\sum_{j=1}^{j=n} A_{ij}X_j = 0, \ (i = 1, 2, 3, \cdots, n)$$

and thinking of n as a thousand million. The connections among the variables in human-relation fields are probably not of this particular kind, but are quite likely capable of as simple and compact statement if the experts in these fields were able to express them in terms of mathematics.

There has recently been organized at Princeton an endowed Institute for Advanced Study, post-graduate in character, under the directorship of Abraham Flexner whose book "Universities, American, English and German," startled the American educational world in 1930 with its severe arraignment of American universities. Dr. Flexner has spent the last two years in preparatory investigations and conferences here and abroad and it is no accident that the first six (and up to the time this article is written, the only) members elected to its faculty are all mathematicians. According to the *New York Times* of October 11, 1932,

> Dr. Flexner has anticipated that he will be asked why the institute has chosen mathematics as its first effort, that some will suggest that the Institute is abandoning the world and its ills in order to embark upon a subject of purely theoretical importance.
>
> "Quite the contrary," he says in a statement prepared for the trustees, "Mathematics is perhaps the most fundamental of all the disciplines. An institution such as we are engaged in establishing need at no time ask of itself what of practical importance can be expected of this or that person, this or that subject, this or that investigation. The history of civilization proves abundantly that sound scientific work does not only gratify human curiosity but almost always leads to unexpected results."

Some students of history think they discern in the story of human progress evidence that great movements in world enlightenment recur in cycles of four hundred years and that we are about to witness the initiation of such a remarkable era. May be, impressed by the wonderful work of Einstein, Eddington and their co-workers, scientific men, trained in inductive methods, are about to employ also and with greater power the mathematical method these men have revived—that older method of deduction, whose use by the ancient Greeks led them to intellectual conquests which never fail to awaken the admiration of all who read the record.

The Jabberwocky Was a Lark or The Mathematician Takes a Holiday

By Joseph Jablonower
Fieldston School, of the Ethical Culture Schools

"You seem to be clever at explaining words, Sir," said Alice, "would you kindly tell me the meaning of the poem called 'Jabberwocky'?" "Let's hear it," said Humpty Dumpty, "I can explain all the poems that ever were invented —and a good many that haven't been invented yet."

When Humpty Dumpty turns his perch on the wall into a lecturer's platform and proceeds to popularize the highly technical "Jabberwocky," he initiates his auditor into only those most obvious of the mathematician's tricks that are exemplified in the poem. He resorts to some of the same dodges on his own account. The present writing is offered as a comment on both the "Jabberwocky" and its commentator—a sort of square root of a negative number. (Eddington, Aldous Huxley, and Bernard Shaw have been setting the fashion of invoking that concept whenever an impossible trick is to be turned.)*

What does Professor Dumpty give Alice by way of elucidation of the text?

"'Brillig' means four o'clock in the afternoon—the time when you begin broiling things for dinner." "That'll do very well," said Alice, "and 'slithy'?" "Well, 'slithy' means lithe and slimy."

And so on, in the same vein, and not very much of it.

There is more to the "Jabberwocky" than this telescoping of words to which Dr. Dumpty confines himself. He was probably reserving a good deal for a later lecture. This lecture may have been the first of a 15-hour course, or he may have had an eye, as do so many of his fellow lecturers, to royalties on a book which he was planning to build around his "reservations." I am above such considerations, and the publishers would not use the "stuff" anyway. So I proceed to give what H. D. would have given, some time or

* Eddington: Nature of the Physical Universe; Huxley: Point Counterpoint; Shaw: Adventures of the Black Girl in her Search for God.

other, in some form or other, if he had not tumbled from the wall. (King's horsemen were no longer so reliable as they had been in the good old days!)

Alice had the instinct of the uninitiated and the unspoiled as she puzzled over the poem for herself. (This was before she had registered for Professor Dumpty's course—she needed the credit.)

> *"Somehow it seems to fill my head with ideas—only I don't exactly know what they are! However, somebody killed something: thats' clear at any rate—"*

You see, she was not leaving all of the reflecting to the looking glass. Had she not been interrupted by so many and sudden interruptions she might—who knows?—have gone on in some fashion such as this:

> *"s R p is a propositional function, where R is known to be the relation 'to be killed.' The proposition is either true or false, a particular s and a particular p serving to make the proposition report a fact or describe an imaginary event."*

The Jabberwocky is replete with instances of such propositional functions.

Consider "Beware the Jubjub bird." The table of values in the propositional formula follows:

s = you (the auditor whom whosit is addressing)
R = beware
p = Jubjub bird.

Dr. H. Dumpty did not get beyond the first stanza. He probably intended to leave the remaining ones for Alice as an exercise. I follow his example.

The reader—if he is still awake and also still with me—is undoubtedly eager now to know what I meant by the insinuating reference to Humpty's very own mathematical dodges. It is his coining of words in order to compact ideas. (Compact *can* be used as a verb.) For example:

> *"I mean by 'impenetrability' that we've had enough of that subject, and it would be just as well if you'd mention what you mean to do next, as I suppose you don't mean to stop here all the rest of your life."*

Mathematicians are like that. When they use words they know just what content they have in mind for them. In one book on Analytic Functions of a Complex Variable we find:

If x takes on in succession an infinite sequence of $x_1, x_2, x_3, \cdots$, we say the variable x has the limit X on the sequence, or that the sequence has the limit X, if the numerical difference $|x_n - X|$ becomes and remains, as n increases, less than any positive number, no matter how small. . . . The definition contained in the opening sentence (the one just given) of this paragraph, when more precisely worded, states that no matter what positive number ϵ, small or large, is proposed, an integer m exists for the ϵ such that all the terms of the sequence beginning with the mth lie in the interval $[X-\epsilon,\ X+\epsilon]$; or, in mathematical notation

$$|x_n - X| < \epsilon, \text{ for all } n \geqq m.$$

Alice's criticism of Humpty Dumpty's definition does not, of course, apply to the definition we have just cited. She remonstrates with her lecturer:

> *"The question is," said Alice, "whether you can make words mean so many* different *things."* [Roman type mine.]

She might also have protested at the way in which he uses in a very technical sense words that are already rich in connotation among the lay people. Compare, however, his technical use of the word "glory" with the physicist's use of the term "work." (Again, I leave this as an exercise for the reader.)

Humpty Dumpty's rejoinder to Alice's remonstrance is not without significance:

> *"The question is," said Humpty Dumpty, "which is to be master, that's all."*

And this should be remembered in his favor:

> *"When I make a word do a lot of work like that, I always pay it extra."*

Humpty Dumpty is a wag. He might have qualified as a teacher, particularly as a mathematics teacher.

> *"Why do you sit out here all alone," said Alice . . .*
> *"Why, because there's nobody with me!" cried Humpty Dumpty.*

At another stage of the colloquium:

> *"How old did you say you were?"* [said H. D.] *Alice made a short calculation and said, "Seven years and six months."*
> *"Wrong!" Humpty Dumpty exclaimed triumphantly. "You never said a word like it."*
> *"I thought you meant 'How old are you'?" Alice explained.*
> *"If I'd meant that, I'd have said it," said Humpty Dumpty.*

Humpty Dumpty's refusal to understand Alice is without malice. He is a good egg, on the wall. [*sic*!]

Not so with some of his fellows among Alice's dream people. In fact, their behavior makes her dreams something akin to nightmares. The Caterpillar, for example:

> *"What size do you want to be," it asked.*
> *"Oh, I'm not particular as to size," Alice hastily replied, "only one doesn't like changing so often, you know."*
> *"I don't know," said the Caterpillar.*

The Caterpillar was quite a cat. He might have passed without comment Alice's use of the stop gap "you know." We all mark time, on occasion, in our ratiocination, you know. The gaps, while they do not promote understanding, do not hinder it and are, at worst, harmless.

Of the same uncharitable stripe was the Cheshire Cat, for all its wide grin.

> *"Would you please tell me which way I ought to go?" [asked Alice.]*
> *"That depends a good deal on where you want to get to," said the Cat.*
> *"I don't much care where . . ." said Alice.*
> *"Then it doesn't matter which way you go," said the Cat.*
> *". . . So long as I get* somewhere," *Alice added as an explanation.*
> *"Oh, you're sure to do that," said the Cat, "if you only walk long enough."*

It is our duty, of course, when we are trying to communicate a thought, or, for that matter, to ask a question, to make our meaning as clear and as unequivocal as possible. But how clear is clear, and how unequivocal is unequivocal? Mathematicians and other good scientists define their terms as well as they are able and use them only in the sense described in the definitions throughout the presentation of their theses. But even scientists demand of their readers a certain amount of "give." The Cat need not have been such a cat! It might have inferred, as it finally did understand, that Alice wanted to know something about the neighbors.

Fruitful intellectual intercourse is engaged in only by those persons who aim to express themselves clearly and who try to understand their fellows generously. The shyster lawyer in the courtroom and the trick debater on the platform make one a place where justice is mocked and the other a place where intellection is travestied.

Particularly is it one of the occupational diseases to which teachers are subject: the disease of refusing to understand what is implied in a statement, to deny credit for unexpressed ideas, to insist on the literal interpretation of word and statement. (Was

the teacher of Mathematics, Charles M. Dodgson working off the poison in his system?) At first, the teacher takes on this uncharitable character for the "good" of his pupil. He wishes to discourage the pupil's tendency to carlessness in thought and speech. But often the professional attitude becomes second nature, and the teacher becomes intellectually stingy.

But we are moralizing. And the Jabberwocky was, after all, just a lark.

Guillaume Francois Antoine de l'Hospital Marquis de St.-Mesme

Born in Paris, 1661
Died in Paris, February 2, 1704

L'HOSPITAL[1] seems to have been interested in mathematics from an early age. It is said, for instance, that when fifteen he solved a problem set by Pascal. He served for a time as a cavalry officer, but he soon resigned from the army and returned to Paris. There his money and social position gave him many advantages and he knew or corresponded with many of the mathematicians of his day. It was in a letter to l'Hospital that Leibniz gave his mathematical creed,—"One of the secrets of analysis consists in the characteristic, that is, in the art of skilful employment of the available signs, and you will observe, Sir, by the small enclosure that Vieta and Descartes have not known all the mysteries."[2]

In 1691, Jean Bernoulli spent several months with l'Hospital teaching him the newly invented calculus which as Ball notes was then really understood only by Newton, Leibniz, and the two Bernoullis.

In 1696, l'Hospital published his *Analyse des infiniment petits, Pour l'intelligence des lignes courbes*, a work on the differential calculus which brought Leibniz's differential notation into use in

[1] The name was later spelled l'Hôpital, the circumflex denoting the omission of the 's.'

[2] F. Cajori, *History of Mathematical Notations*, II, p. 185.

France and which helped to make the calculus known in Europe. In the preface, the author discusses the parts taken by Leibniz and the Bernoullis in the development of the calculus passing over Newton's contribution with a casual reference and dwelling on the importance of the notation used by Leibniz. The idea of infinitely small quantities seized people's fancy and it is said that a song about "les infiniments petits" became very popular.

This book was reprinted in 1716, 1720, and 1768. In 1730, Edmund Stone published a translation in English supplemented by a work on the integral calculus, the title being, *The Method of Fluxions both Direct and Inverse. The Former being A Translation from the Celebrated Marquis De L'Hospital's Analyse des Infiniments Petits* (sic); *And the Latter Supply'd by the Translator.* It is interesting to note that the translator used the Newtonian symbolism and he seems to have generally amended the text to the Newtonian point of view. The discrepancies which arose when the word "différence" was translated as "fluxion" are described in Cajori's *History of the Conceptions of limits and Fluxions in Great Britain from Newton to Woodhouse*, p. 50 ff. Jean Bernoulli seems to have thought l'Hospital guilty of plagarism but Leibniz replied to the accusations point by point. In l'Hospital's defence, it should be noted that he admits his indebtedness to Leibniz and to the Bernoulli's especially to Jean, a thing that many plagarists omit doing.

L'Hospital's other important work was a treatise on conics which appeared posthumously in 1707 and which became, according to Ball, the standard work for over a century. This was translated into English by Stone (1723).

Whatever l'Hospital's merit as an independent worker, it must be conceded that the publication of these two treatises both of which were influential for a century was no mean achievement.

VERA SANFORD

Be sure to look for the announcements of the meetings of mathematicians in Chicago on pages 311-312.

NEWS NOTES

THE following program was rendered by the Detroit Mathematics Club this year:

December 8: "Mathematics and Its Relation to Engineering," Russell E. Lawrence, Lawrence Institute of Technology, Jefferson Intermediate School.

January 19: "Mathematics in the Secondary School," Dr. Everett L. Austin, Michigan State College, Eastern High School.

February 9: "Research in Mathematics," Dr. Clifford Woody, University of Michigan, Sherrard Intermediate School.

March 9: "Synchronizing Mathematics Teaching with Business Needs," James Lobban, Detroit Edison Company, Durfee Intermediate School.

Officers: President, Jas. J. Cruikshank; Vice-President, Paula Henze; Secretary, Clara Mueller; Treasurer, James B. Mount.

Program Committee: May F. Walsh, Helen M. Wattles, Duncan Pirie.

Joint Sessions of the Mathematics and Arithmetic Sections of the California State Teachers Association were held in Oakland and San Francisco on November 23, 1932.

Wednesday, November 23, 9:30 A.M. Auditorium, Franklin School, Oakland:

Chairman: Jean Tuttle, Roosevelt High School, Oakland.

"The Articulation in Mathematics From Kindergarten Through High School," Sophie H. Levy, Assistant Professor of Mathematics, University of California.

"The Fourth Dimension," Derrick Norman Lehmer, Professor of Mathematics, University of California.

Business Meetings for Election of Officers.

Wednesday, November 23, 1:30 P.M. Room 30, High School of Commerce, San Francisco:

Mathematics Section: Chairman, Irene Pauly, Balboa High School, San Francisco; Secretary, Maud Volandri, Balboa High School, San Francisco.

Arithmetic Section: Chairman, Clara H. White, Principal, Laguna Honda School, San Francisco.

Business Meeting Mathematics Section: Election of Officers.

Vocal Solos: Aileen Hennessy, High School of Commerce, San Francisco.

"The Articulation in Mathematics From Kindergarten Through High School," Sophie H. Levy, Assistant Professor of Mathematics, University of California.

"The Prediction of Success in First Year Algebra," Lillie Lewin, Assistant, Bureau of Educational Research and Service, San Francisco.

"The Fourth Dimension," Derrick Norman Lehmer, Professor of Mathematics, University of California.

Business Meeting: Arithmetic Section: Election of Officers.

The Association of Mathematics Teachers of New Jersey held its forty-sixth meeting at Atlantic City on Nov. 12th, 1933. The program follows:

"The New State Course of Study in Mathematics," Dr. Frank J. McMackin, Dickinson High School, Jersey City.

"Mathematics for the Better Student," Professor William R. Longley, Yale University.

The forty seventh meeting of the above Association was held at the Burlington (N. J.) High School on March 11, 1933. The program follows:

Morning Session:

"Some Functions in Higher Mathematics," Dr. Emory P. Starke, Rutgers University.

"The Unifying Principle in the New State Syllabus—The Function Concept," Mr. William W. Strader, Dickinson High School, Jersey City.

Afternoon Session:

"The Teaching of Ninth Grade Mathematics—The New State Syllabus," Dr. Howard F. Hart, Head of Mathematics Department, East Orange High School.

"Individual Differences in Ninth Grade Mathematics—The New State Syllabus," Mr. H. P. Collins, Principal, Junior High School No. 1, Trenton.

Officers of the Society: Virgil S. Mallory, President, State Teachers College, Montclair, N. J.: Miss Amanda Loughren, Vice-President, Thomas Jefferson High School, Elizabeth, N. J.; Andrew S. Hegeman, Secretary Treasurer, Central High School, Newark, N. J.

Council Members: The officers and Dean Luther P. Eisenhart, Professor Charles O. Gunther, Professor Richard Morris, Harrison E. Webb, Frederic J. Crehan, Harold I. Palmer, Roscoe P. Conkling.

The Association of Teachers of Mathematics of the Middle States and Maryland held their Thanksgiving meeting in Atlantic City on Friday Nov. 25th, 1932. The program follows:

Business Meeting with Election of officers.

"This Simple Universe," Bancroft H. Brown, Assistant Professor of Mathematics, Dartmouth College.

"A New Plan for Eleventh and Twelfth Grade Mathematics," Vevia Blair, Teacher of Mathematics, Horace Mann School for Girls.

Discussion.

The officers of the Association are: President, Mr. C. A. Ewing, The Tome School, Port Deposit, Maryland; Secretary, Miss Ruth Wyatt, Woodrow Wilson Junior High School, Philadelphia.

The Mathematics Section, California Teachers' Association, Bay Section, which is a member of the National Council of Teachers of Mathematics, elected the following officers for the year 1933 at their annual fall meeting:

Chairman: Mr. James W. Hoge, Supervisor of the Teaching of Mathematics at University High School, Oakland, Calif.

Vice-Chairman: Mr. S. A. Francis, Instructor of Mathematics at San Mateo Junior College, San Mateo, Calif.

Secretary: Mrs. Nina H. Slewing Supervising Teacher of Mathematics at University High School, Oakland, Calif.

The Men's Mathematics Club of the Chicago and Metropolitan area held its third meeting of the season on January 20, 1933. The following program was given:

"The Lag of Mathematics Previous to the Seventeenth Century," Professor H. E. Slaught, University of Chicago.

"The Algebraic Formula," Professor O. F. Rusch, Concordia College.

The December meeting of the club set a new record for attendance and interest. All members were presented with copies of W. W. Gorsline's new book on the Slide Rule. Talks by Dr. Stone, Marx Holt and Mr. R. E. Hughes were received with marked interest.

Mr. F. W. Runge, 1421 Wesley Ave., Chicago, Ill. is the secretary of the club.

The Mathematics Club of the New Jersey State Teachers College at Montclair has the following officers and program for the year 1932–33: President, Verna Ridsdale; Vice President, Eleanor Hagen; Secretary, William Dunn; Treasurer, Clara Jillard. Program:

October 8, Business.

October 22, Dr. Kramer, "Mapping In the Complex Plane."

November 9, Miss Mildred Thompson, "Great Men In Mathematics."

November 20, Mr. I. Megibow, "Higher Conceptions of Conic Sections."

December 3, Mr. Urban Barrett, "Extracting Cube Roots."

January 12, Miss Margaret Dunn, "Projective Geometry."

January 25, Miss Ethel Bellis, "Mathematical Recreations."

February 16, Miss Helen Anderson, "Theory of Numbers."

March 9, Miss Sarah Bogert, "Theory of Numbers."

March 21, Mr. Ernest Panucci, "Map Projections."

April 4, Miss Edith Day, "Mathematical Knots."

April 18, Social—Bridge.

May 11, Mr. Harrison Webb, "Duplex Algebra."

May 25, Miss Sara Goodman, "Stars of the Summer Sky."

June 8, Mr. Howard Fehr, "Calculus For the Twelfth Grade."

The second dinner meeting of Section 19 (Mathematics) of The New York Society for the Experimental Study of Education was held on Nov. 19th, 1932 at the Men's Faculty-Club of Columbia University. Miss Alberta Welch the retiring chairman of the Mathematics department at Bryant High School was the honor guest and spoke on "How it Feels to be Retired."

Professor David Eugene Smith spoke on "Recent Experiences at the International Congress of Mathematicians at Zürich" and Professor C. B. Upton of Teachers College, Columbia University, spoke on "The Dow Theory of Averages."

At the third dinner meeting Miss Meta Wood of the Lincoln School spoke on "The Teaching of Mathematics in the Secondary Schools of Germany."

The Winthrop College Branch of the National Council of Teachers of Mathematics (Rock Hill, S.C.) held a regular meeting, Tuesday, December 6, 1932, with Miss Alice Ann Grant, the president, presiding. Miss Fannie Watkins, Chairman of the Program Committee; Miss Kate Rosen, chairman of the Publicity Committee; Miss Sarah Mae Hester, chairman of the Absentee and Excuse Committee; Miss Mildred Stukes, chairman of the Social Committee; Miss Louise Weill, chairman of the Membership Committee, each gave a report of the work done and decided upon by their respective committees. The kind offer to the Branch by Dr. G. T. Pugh, Head of the Mathematics Department, and Mrs. Pugh of the use of their home for the Christmas meeting was enthusiastically accepted.

The meeting held at the home of Dr. and Mrs. Pugh took the form of a Christmas party. Games, a contest from "The Mathematics Teacher" led by Miss Hortense Rogers, singing of Christmas carols and refreshments were enjoyed. The singing was accompanied by Miss Catherine Allgood on the violin with Miss Alice Ann Grant at the piano. Dr. Pugh acting as Santa Claus, gave to each Junior and Senior in Mathe-

matics and each member a copy procured by Miss Grant, of C. J. Keyser's "Mathematics as a Career." Each one present received the good wishes of the Branch done up in compact form—compactness being one of the factors making mathematics such a force in the world.

CAM RHODES RAWLINSON, *Local Editor*

The regular meeting of the Winthrop College Branch of the National Council of Teachers of Mathematics (Rock Hill, S.C.) was held January 3, 1933. The engraved charter of affiliation of the Winthrop College Branch of the National Council of Teachers of Mathematics was shown to those present and Dr. Pugh offered to have it framed.

The program was as follows:

"Getting Acquainted with the Mathematical Library"

A. The Rhind Papyrus
   1. Story of the Rhind Papyrus—Catherine Allgood
   2. Publication—Nelle Douglas
   3. Review of Volume I—Elizabeth Harmon
   4. Review of Volume II—Elizabeth Jones

B. Our Mathematical Library
   1. History—Thelma Robinson
   2. Philosophy—Willie Faye Taylor
   3. Recreation—A. Roberta Wooten.

At the regular meeting of the Winthrop College Branch of the National Council of Teachers of Mathematics, the program was as follows:

"Getting Acquainted with the Mathematical Library"

C. The Yearbooks of the National Council of Teachers of Mathematics
   1. The First Yearbook—Sara Mae Hester, Kate Rosen
   2. The Second Yearbook—Willie Faye Taylor
   3. The Third Yearbook—Lillian Henderson
   4. The Fourth Yearbook—Catherine Allgood
   5. The Fifth Yearbook—Louise Weill, Elizabeth Harmon Carolyn Snipes
   6. The Sixth Yearbook—Mildred Stukes, A. Roberta Wooten
   7. The Seventh Yearbook—Worthe Rowland

CAM RHODES RAWLINSON, *Local Editor*

According to the President, Mr. E. D. Lewis of Evander Childs' High School, New York City, of the National Education Association, the department will hold two general sessions on Monday, July 3rd and Wednesday, July 5th at 2 P.M. and a series of Round Table Conferences on Monday, July 3rd at 3:30 P.M. at the Stevens Hotel in Chicago, Illinois. The program follows:

FIRST GENERAL SESSION

1. Report of the Joint Committee on the Emergency in Education—James B. Edmonson, School of Education University of Michigan.

2. "The Changing High School Curriculum"—Leonard V. Koos, School of Education, University of Chicago, Director of the National Survey of Secondary Education.

3. Discussion—Joseph B. Orleans, George Washington High School, New York, N. Y. and Chairman of the Committee on the Integration of High School Studies of the Department of Secondary Education.

SECOND GENERAL SESSION

"Relations between the College and the Public High School"—P. Roy Brammel, School of Education,

Connecticut Agricultural College, Storrs, Connecticut, and Specialist in the National Survey of Secondary Education.

Jerome Kerwin, Dean of the School of Education, University of Chicago.

George Works, University of Chicago.

Leader of Discussion: Mary E. Courtnay, Dean of Girls, Lindblom High School, Chicago, Illinois.

ROUND TABLE CONFERENCE IN MATHEMATICS—IN COOPERATION WITH THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Chairman: E. C. Hinkle, Chicago Normal College, Chicago, Ill.

"Mathematics in the Changing Curriculum of the High School." Edwin S. Lide, Specialist in the National Survey of Secondary Education.

Leader of Discussion: Wm. Betz, Department of Mathematics, Alexander Hamilton High School, Rochester, N.Y. and President of the National Council of Teachers of Mathematics.

Following the Round Table Conference in Mathematics on July 3rd there will be a joint dinner meeting of the Mathematics Section of the Central Association of Science and Mathematics Teachers and the National Council of Teachers of Mathematics at which Dr. C. A. Stone the president of the Central Association of Science and Mathematics Teachers will preside. Professor W. W. Hart of the University of Wisconsin will discuss "Mathematics from the Standpoint of the Educator," Dr. H. E. Slaught of the University of Chicago will speak on the topic "Retrospect and Prospect for Secondary Mathematics," and a third speaker to be chosen will speak on "Applied Mathematics—Necessary in the Social and Physical Sciences.

The price of the dinner will be $1 and the place the Great Northern Hotel. These meetings may well turn out to be epoch making and all those teachers interested in the welfare of mathematics should attend these meetings if it is at all possible.

The Mathematics Teacher

Perhaps you teach about the lovely things of earth
Of doctrines old, or doubtful ones of recent birth;
But e'en a nobler work I think I'm called to do
Because I teach of things which are forever true.

Mine not an easy task—to help the youthful soul
Through darksome rugged ways, to reach Truth's starlit goal;
And yet along the sometimes weary way I plod
To find unchanging truth forever leads to God.

Sister M. Gertrude, O. P., Our Lady of Mercy
Boys' School, New York, N.Y.

NEW BOOKS

Orleans and Hart's Intermediate Algebra. By Joseph B. Orleans, and Walter W. Hart. Pages VII+301, D. C. Heath and Company, 1933. Price $1.32.

This new text for classes in second-year algebra is intendent to meet the requirements of the College Entrance Examination Board and the New York Regents. The arrangements of topics, the method of presentation, and the proportioned treatment reveal a book planned to be of assistance for the efficient teaching of algebra and, with regard to the student, a book planned for a logical, practical and pleasant study course.

There are reviews throughout. The student comes to second-year algebra usually after several months, or even a year, of separation from the first-year work. The introductory chapter is a clearly indicated review of elementary algebra in the form of a series of *thirteen diagnostic tests* followed by *remedial instruction and practice exercises.* While doing this review work the student is also supposed to make progress in second-year work, as new material is introduced in this chapter because of its logical connection with contiguous topics. The second chapter is devoted to a review of the ordinary types of problems taken up in the first year. Almost every chapter is followed by a *mastery test.* There are also four *cumulative reviews* at appropriate points, and a *general review* at the end, containing over 200 problems, some of which have been taken from College Entrance Examination papers.

The authors have tried to unify the course and to give it a practical significance for the pupil by an early introduction and continued stress on the *functional relation.* Also, further to secure unity and natural sequence, *logarithms and trigonometry* are placed relatively early in the course; *variation* is taken up immediately after *quadratic functions of two variables*, and is given a judiciously extended treatment.

There is a total of *474 verbal problems.* To an unusual extent the authors have drawn these problems from fields already familiar to the student.

Graphing of first and second degree functions is presented and the resulting graphs are thoroughly discussed.

The *Additional Topics* comprising the fourteenth and last chapter extend the basic course for classes which are able to do more than the minimum course, and provide a means of meeting the problem of *individual differences* in pupil ability. The last of the additional topics is a unit of elementary *Statistics*, the value of which in connection with the social studies especially is now being pointed out.

The Teachers Many Parts. By Sir John Adams. Pages 351. Ivan Deach, Jr., 1932.

This book is one of a long list by the author dealing with various phases of education. It deals with "The Outer World–Public," "The School–Professional," and "The Inner World–Personal."

In these days when so much is being said about the importance of the teacher

knowing a little beyond his own narrow field, one can find much of interest and of value in this new book.

The aim of the book is a friendly stock-taking among professional colleagues. It was written as a sort of personal interview between the author and whichever of his fellow teachers he was able to get interested enough in his subjects to carry on a silent conversation.

Any omissions made by the author are to be supplied by the reader.

A Short Course in Trigonometry. By James G. Hardy. Pages IX+137. The Macmillan Co., 1932.

This textbook is the result of more than thirty years of experience in teaching trigonometry to college students whose maturity is advanced enough to satisfy any desire on their part to know what trigonometry is and what it does.

The purpose of each topic presented is discussed and the author attempts to set forth the subject as a continuous development closely connected with other branches of knowledge. Numerous and varied exercises appear not all of which demand algebraic skill, but are intended to develop insight and real understanding.

Analytic Geometry. By F. S. Nowlan. Pages XI+289. McGraw-Hill Book Co. 1933. Price $2.25.

The authors planned this textbook primarily for the use of first- and second-year university students. It is the result of many years' experience in teaching the subject in American and Canadian Universities. The book is intended to contain subject matter which, while easily understood by the student, will at the same time be analytic in character and strictly rigorous.

Some of the distinguishing features of the book are (1) The slope of a line is defined in such a way as not to presuppose a knowledge of trigonometry (2) Extensive use is made of orthogonal projection, as thereby generality of demonstration is simply obtained, and (3) The study of conics is based upon a definition of the general conic.

Freshman Mathematics. By H. L. Slobin and W. E. Wilbur. Pages XVIII+425. Ray Long and Richard R. Smith Inc. 1932.

This book is a course in the Essentials of algebra, trigonometry, and analytical geometry the purpose of which is to meet the needs of "average students from average secondary schools, for an adequate foundation of mathematics irrespective of the students' general objectives."

The three subjects mentioned above are presented in tandem fashion in order to furnish adequate drill and general preparation in each subject, and to build an adequate accumulative preparation for the infinitesimal analysis.

The course is limited to the essentials and deviates from the usual course mainly in the selection and arrangement of the material which is a good point to keep in mind.

Elementary Mathematical Analysis, Vol. II. By Mayme Irvin Logsdon. Pages IX+185. McGraw-Hill Book Co. 1933. Price $1.75.

This is the second volume of a freshman text in mathematics, the two volumes affording complete preparation for the formal course in calculus usually given in the sophomore year.

This volume continues and extends certain topics begun in the first volume and include any additional topics which are to be used later in the study of the calculus.

The books are well written and printed and constitute a very interest-

ing course of study for teachers who wish to be more progressive in their methods of teaching.

Teachers of mathematics will wish to see these two interesting volumes.

Mathematical Excursions. By Helen Abbot Merrill. Pages XI+143. The Norwood Press. 1933.

The subtitle of this book is "Side trips along paths not generally traveled in elementary courses in mathematics." The book contains a large collection of problems which the author gathered for her own use in the class room, and is not written for learned people.

Anyone interested in problems the solution of which involves only some of the simplest types of algebra and geometry will find something here to prove diverting, to furnish mental exercise and to point the reader toward other things that may be found further along the mathematics road.

Outline of The History of Mathematics. By Raymond Clare Archibald. The Society For The Promotion of Engineering Education. 1932. Pages 53. Price 30 cents.

This pamphlet is number 18 of a list of Selected Papers of the Summer School for Engineering Teachers which have been published and may be obtained from the Lancaster (Pa.) Press. The first part of the pamphlet deals with the history of mathematics before 1600 A.D. and the second part with the history of mathematics after 1600.

The author refers briefly to some developments of the nineteenth century in connection with topics usually discussed at undergraduate colleges.

New Algebra for Schools. By C. V. Durell. Vol. I, Parts I and II. Pages IX+328. Vol. II, Part III. Pages XIII+198. G. Bell and Sons. 1932. Prices Vol. I, 4s.6d, Vol. II, 3s.6d.

These two volumes are issued in two styles (1) Without appendix and with answers and (2) With appendix and with answers. Parts I and II are also available separately, without appendix and without answers.

In Parts I, II and III the author attempts to bring together and use fully the many detailed suggestions on methods of teaching algebra, choice of subject-matter, selection of material for illustrative purposes, construction of exercises and test-papers, and the like which he has made a part of his work for the last fifteen years.

Part I deals with notation, formulas, simple equations and problems; Part II includes factors, fractions, simultaneous and quadratic equations; Part III completes the course for "additional mathematics" in School Certificate; and the concluding volume, Part IV (not yet received) contains Higher Certificate work, stopping short of the requirements for "distinction papers."

Teachers in American schools will find much interesting material of value in these volumes.

Graph Book. By C. V. Durell and A. W. Siddons. Pages VIII+80. G. Bell and Sons. 1929. Price 2s.6d.

This is a teacher's edition with hints and answers. The purpose of the book is to set out in detail a course which will utilize to the best advantage whatever time is available for graphical study.

The size of the page makes it possible for the illustrative examples to be shown on a working scale instead of being reduced as in the ordinary text book. Moreover, since the work of the pupil is to go in the book his work may be preserved for future reference. American teachers will find this book of interest.

Introduction to Practical Astronomy. By Dinsmore Alter. Pages VIII+80. Thomas Y. Crowell Co., 1933. Price $2.

This book covers a field entirely different from that of the ordinary textbook on astronomy. In its general makeup it is a laboratory manual; as to its content it serves as a textbook on practical astronomy. This combination helps to make the book serve the pupil better than the traditional textbook which often presupposes laboratory experience.

Graduated Exercises in Elementary Mathematics. By E. N. Mozley. Pages VIII+51. G. Bell and Sons. 1s.3d.

This little pamphlet consists of graduated exercises the purpose of which is to furnish a course, by means of examples only, covering the entire ground for School Certificate (excluding the additional mathematics papers) and for Scholarship Examinations into Public Schools.

No attempt is made to give instruction or advice as to how to proceed as the author holds that pupils learn little rom the methods taught in books.

Number—The Language of Science. By Tobias Dantzig. Pages VII+255. The Macmillan Company, 1933.

This is the second edition, revised, of a book which every teacher of mathematics should read. It is an important addition to the library of scientific books for the busy reader. The author attempts to bridge the gap between the vague perception of number as it appears to the primitive savage and the more or less involved number concept which forms the basis of modern science.

Under such headings as *Finger-prints*, *The Empty Column*, *Number Lore*, *This Flowing World*, and *Reality and Fiction* he gives us information without a large number of mathematical technicalities.

Plane Geometry and Solid Geometry. By A. M. Welchons and W. R. Krickenberger. Pages XIV+370 (Plane). Pages XII+251 (Solid) Ginn and Co., 1932. Prices $1.28 and $1.24 respectively.

These two new textbooks in plane and solid geometry are the work of two experienced and successful teachers of mathematics who have attempted to conserve the best traditional practice but at the same time have introduced what they thought desirable and worth while from the more progressive tendencies in teaching geometry.

The special features of the books include such items as *Provision for Individual Differences in Ability*, *Proper Stress on Theorems*, *Problems and Exercises*, *Summary of Methods of Proof*, *Treatment of Locus*, Treatment of Numerical Trigonometry, Tests and the like. The books are good looking, they contain many of the more modern features of good geometry textbooks and will be of interest to teachers of mathematics who are looking for more modern material.

The Teaching of Mathematics in the Secondary School, The Eighth Yearbook of The National Council of Teachers of Mathematics. 264 pp. Bureau of Publications, Teachers College, Columbia University, 1933. Price $1.75.

In 1926, the National Council of Teachers of Mathematics began the publication of a series of Yearbooks. That First Yearbook, a paper bound affair with pages smaller than those in later yearbooks, was entitled, "A Survey of Progress in the Past Twenty-Five Years." It is now out of print. Later Yearbooks have carried titles as follows:

2. Curriculum Problems in Teaching Mathematics.

3. Selected Topics in the Teaching of Mathematics.

4. Significant Changes and Trends in the Teaching of Mathematics Throughout the World Since 1910.

5. The Teaching of Geometry.

6. Mathematics in Modern Life.

7. The Teaching of Algebra.

All of these, except the second, are available at the same price as that indicated for the Eighth Yearbook, $1.75. The Second Yearbook, bound in paper, may be obtained for $1.25.

It will be seen that the mathematics teacher whose private library does not contain these volumes can obtain the entire seven yearbooks for $11.75. The reviewer owns all eight of these yearbooks and considers them well worth their cost.

A wide variety of topics is discussed by the six contributors to the Eighth Yearbook. Professor Breslich of the University of Chicago contributes the first chapter, taking as his topic, "Articulation of Junior and Senior High School Mathematics." He points out very effectively the responsibility of teachers in both the junior and the senior high school for planning a course which will be a continuous affair. This continuity will apply to arithmetic (too often disposed of before the senior high school is reached) as well as to algebra and geometry. "An ideal situation is to have but one department and one supervisor to direct it" (p. 2). This is Professor Breslich's third contribution to this series of yearbooks. Others will be found in the Third and the Seventh Yearbooks.

The second chapter is entitled, "A Summary of Some Scientific Investigations of the Teaching of High School Mathematics." It has been written by Professor Benz of Ohio University. From a large mass of material appearing in book and periodical form, the author has selected 131 titles which may be classified as scientific investigations, has analyzed and classified them and has organized them into a report which, after the introduction, is presented in eight sections. This chapter should be of great value to graduate students and others who desire to survey the scientific literature of a portion of the field of mathematics before undertaking investigations of their own. This chapter will remind the reader of the valuable summaries of investigations in arithmetic by Judd and Buswell and in reading by Gray.

By far the longest, and in some respects the most valuable, contribution in the Eighth Yearbook has been written by William Betz of Rochester, President of the National Council of Teachers of Mathematics. The subject of Mr. Betz's contribution is, "The Teaching of Intuitive Geometry, a Study of Its Aims, Development, Content, and Method." This chapter is a scholarly and thorough discussion of intuitive geometry as to its educational significance, its development as a school subject in this and in foreign countries, its basic content, the organization of its instructional materials, and suitable classroom methods and devices. Some notion of the thoroughness of the treatment may be gathered from the fact that the chapter contains 153 footnotes, most of which refer to works on psychology and mathematics and some of which are foreign publications not available in English translations. The chapter with its 110 pages is perhaps too long and too detailed to be of maximum service to high school teachers but it is a gold mine for those who wish really to be students of this subject. This is Mr. Betz' fourth contribution to the National Council Yearbooks. His earlier contributions ap-

pear in the First, Third and Fifth Yearbooks.

To the reviewer, the most interesting chapter in the Eighth Yearbook is the fourth. It is called, "Coherence and Diversity in Secondary Mathematics, a Personal Philosophy Based on the Opinions of Many Leaders in Education." This chapter has been prepared by Professor Beatley of Harvard University. A very interesting discussion of the kind of mathematics which should be offered to pupils if varying intellectual capacities is accompanied and supported by the returns from a questionnaire circulated to a select list of general educators, and college and high school teachers of mathematics. The returns from these groups showed "a surprising agreement." The chapter culminates in a schematic representation of a proposed course for superior students and for inferior students for each of the six high school years. Professor Beatley's earlier contributions will be found in the Second and the Fifth Yearbooks.

The one contributor from a foreign country is Georg Wolff of Herschelschule, Hanover, Germany. Mr. Wolff writes on "The Mathematical Collection." This chapter will supply the American high school teacher with very valuable suggestions on concrete aids in teaching mathematics and incidentally, will indicate rather forcefully the superior work done in the *Oberrealschulen* of Germany. To aid in the teaching of the parabola, for instance, a most interesting array of devices is pictured. If one may judge from Mr. Wolff's account of the contents of their equipment room, the American high school lags considerably behind the German *Oberrealschule* in its collection of mathematical aids and devices.

The sixth and final chapter is entitled, "Units of Instruction in Secondary Mathematics" and has been prepared by Professor Georges of Crane Junior College, Chicago. This chapter indicates the advantages of units of instruction in contrast to the chapters of the typical textbook. The unit, direct linear variation, is discussed and outlined in detail. Teachers desiring to work in the direction of the unit plan and to provide greater opportunity for the development of the functional concept will find this chapter replete with helpful suggestions. Readers of the Eighth Yearbook will look forward with much interest to further contributions by Professor Georges on this subject.

On the whole, the Eighth Yearbook is a decidedly worthwhile contribution to the literature dealing with the teaching of secondary mathematics. It should be owned by each mathematics teacher in the junior and senior high schools.

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Under the chairmanship of Professor Judd, the Committee on Materials of Instruction of the American Council on Education is issuing a series of pamphlets of which three are now available. These are:

Achievements of Civilization†—(1) *The Story of Writing* (64 pp.); (2) *The Story of Numbers* (32 pp.); and (3) *The Story of Weights and Measures* (32 pp.). The second and third are those with which we are chiefly concerned.

The basic idea is to offer interesting reading units "to show pupils that the subjects which are sometimes thought

† These may be obtained from the committee, 5835 Kimbark Avenue, Chicago, Ill. The price is 20¢ for those of 64 pages, 10¢ for those of 32 pages. Discounts are given on orders of 25 or more. The three noted above were prepared by Bertha M. Parker and edited by May Diehl.

to be abstract and difficult, such as arithmetic, are designed to put them in possession of great intellectual inventions produced by long ages of human co-operation." The booklets are attractive in form and are so priced as to be available to many schools. One wonders why someone did not undertake this long ago.

In commenting on such a series, we should consider the sources from which the material is drawn, the selection of material, and its treatment. The third point can be taken care of simply. The treatment is excellent.

So far as the sources and selection are concerned, it must be agreed that in presenting historical materials to children, it is often necessary to do what Sir Charles Singer has called "expressing a complex truth as a simple falsehood." That is, it is legitimate to shear off the phrases with which an historian guards his statements and word the statements without these qualifying expressions. For instance, it would not be suitable to discuss the theories of the origin of the zero, but it is desirable to do as the author has done and spent some time on the importance of the zero in the Hindu-Arabic number system.

It is probable that the references given in these booklets represent but a small portion of the materials that were consulted. But if the list is supposed to be suggestive of further work for the teacher or as a cross section of representative works, why was Gow's *Short History of Greek Mathematics* (1884) cited instead of Heath's *History of Greek Mathematics* (1922) or his *Manual of Greek Mathematics* (1931). Why was Sterner's *Geschichte der Rechenkunst* (1891) given instead of Tropfke's *Geschichte der Elementar-Mathematik* (1902, 1920 or 1929)? Above all, if the references are to suggest treatments that are both readable and scholarly, why was Professor Smith's *Number Stories of Long Ago* omitted? An article dealing with Jefferson's connection with our weights, measures, and coinage appeared in *Isis* while these pamphlets were in press and so of course could not be consulted, but Adrien Favre's *Les Origines du Système Métrique* (Paris 1931) would have suggested important considerations which the writer does not mention. Favre shows at some length that the metric system was not only the result of the desire of the scientists but that it also had its origin in the economic difficulties due to the diversified units in France and that standardization had made considerable progress in the century before the French Revolution. The metric system owes a debt to the French economists as well as to the scientists.

The booklet on numbers begins with a discussion of the difficulties of using Roman numerals laying the scene in the court of Charlemagne. We are told that Alcuin prepared an arithmetic from which a problem in multiplication is quoted, and from which a verbal problem is given. Now as the *Propositiones ad acuendos juvenes* from which this seems to be drawn cannot be dated with certainty before the tenth century, and as Charlemagne is little known to junior high school students and as Alcuin is wholly unknown, would not the point be better made by reviewing the Roman numerals, telling that they were used in Europe longer than our Hindu-Arabic numerals have been used, and show how hard it is to multiply in that system. It is the difficulty of the work rather than the provenance of an illustration that is important.

Under "counting" no specific mention is made of the use of finger notation as an international sign language by traders in the middle ages. It has been a matter of experience that pupils

are greatly interested in this point.

In discussing the Greek numerals, no mention is made of the Herodianic numerals which presented marked resemblance to the Roman ones. Further, in treating the system which has twenty seven characters, these letters are given in lower case type instead of the capitals which were really used.

It might have been clearer had the discussion of counters preceded the work in the Hindu-Arabic numerals, suggesting that ease of working with counters might have been a potent factor in the slow acceptance of the new numerals in Europe.

Calculation with Hindu-Arabic numerals is said to have been called "algorismus after Algoritmi a famous mathematician of Bagdad." Wouldn't it be better to use the form al-Khowarizmi?

Then, in discussing computation in the early printed arithmetics, the author chooses a method for long multiplication given by Steinmetz (1568) ignoring the more popular and more comprehensible *gelosia* method in which the same principle of writing out the partial products in full without any addition is followed, but in which the work is so arranged as to be readily understood.

In the work on decimal fractions, it would have been interesting to show a facsimile of Stevin's notation.

The work on weights and measures is disappointing in its neglect of the social and economic importance of standard weights. It seems to be true that under a stable and centralized government weights and measures tend to be uniform while under an unstable and decentralized state, measures tend to be diverse. This item and the part weights and measures have had in legislation is ignored except insofar as statutes are quoted to show the determination of the standards. No mention is made of the important clause in Magna Carta or of the parallel statements in the Articles of Confederation in this country.

These comments are given as an expression of regret rather than as a criticism. The pamphlets are good, but they might easily have been much better.

VERA SANFORD

Education does not mean teaching people to know what they do not know. It means teaching them to behave as they do not behave. It is not teaching the youth of England the shapes of letters and the tricks of numbers, and then leaving them to turn their arithmetic to roguery, and their literature to lust. It is, on the contrary, training them into the perfect exercise and kingly continence of their bodies and souls. It is a painful, continual, and difficult work; to be done by kindness, by watching, by warning, by precept, and by praise,—but above all—by example.

John Ruskin